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The trade-off theory revisited: On the effect of operating leverage

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Abstract

This paper investigates the effect of operating leverage, and the subsequent abandonment option available to managers, on the relationship between corporate earnings and optimal financial leverage, thereby providing an alternative (rational) explanation for the observed negative relationship between these two quantities. Working in a dynamic capital structure setting, where corporate earnings are modelled as an exogenous stochastic process, we explicitly add fixed operating costs to the firm's value optimisation. This introduces a degree of operating leverage and a non-zero value to the implicit abandonment option of the firm's manager. Solving for the firm's optimal timing and financing decisions we are able to derive the relationship between current corporate earnings and optimal financial leverage for a large class of earnings uncertainty assumptions. The theoretical implications are then tested empirically using a large selection of S&P 500 firms. Our analysis reveals that the manager's flexibility to abandon the project introduces nonlinearities into the valuation that are sufficient to reconcile the trade-off theory with the empirically observed negative earnings/financial leverage relationship. We further find theoretical and empirical evidence of a positive relationship between operating and financial leverage. Previous studies have used mean-reverting earnings as an explanation for the observed negative earnings/financial leverage relationship in a trade-off theory setting. We show that the relationship does not need to be process specific. Instead, it is a direct result of the financial flexibility of managers.

Keywords: Trade-off theory, operating leverage, financial leverage, abandonment option
JEL classification: G32, G13, D21.

1. Introduction

Since the pioneering work of Miller and Modigliani (1958) the capital structure literature has attempted to solve the "capital structure puzzle" (Myers, 1984). To date, two main theories have emerged providing alternative perspectives on the optimal use of corporate debt. The static trade-off theory (Scott, 1976) assumes that a manager sets a target debt ratio in order to *trade off* the

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benefits (tax advantages) and cost (bankruptcy cost) of debt financing. The more recent pecking order theory is based on informational asymmetries and states that firms prefer internal over external financing, and debt over equity in the case of external financing (Myers, 1984).

Whilst selected implications of the trade-off theory have received empirical support (Frank and Goyal, 2008), in its classic (static) form, the trade-off theory predicts a positive relationship between earnings and leverage (Shyam-Sunder and Myers, 1999); a prediction which appears inconsistent with the well-established empirical evidence of a negative earnings/leverage relationship (Titman and Wessels, 1988, Rajan and Zingales, 1995).[1] The pecking order theory, on the other hand, predicts such an inverse relationship, leading researchers to conclude that this empirical regularity is consistent with the pecking order theory and inconsistent with the trade-off theory.

Due in part to such incongruent empirical evidence the static trade-off theory fell out of fashion and researchers abandoned taxation and bankruptcy costs as the key drivers of capital structure decision making; turning instead to agency conflict (Jensen and Meckling, 1976) and adverse selection (Myers, 1984) explanations. In recent years, however, the static trade-off theory has been revisited and extended to incorporate dynamic features into valuations and financing decisions. Such models have become known as *dynamic* trade-off models.[2] Importantly, these dynamic models depart from their static counterparts in many interesting ways, leading Frank and Goyal (2008, p.194) to claim that "some of the most prominent objections to the trade-off theory have become less compelling in light of more recent evidence and an improved understanding of some aspects of the dynamic environment."

This paper develops such a dynamic trade-off model and attests to the ability of the dynamic trade-off theory to explain the observed leverage/earnings relationship. More specifically, we revisit the trade-off theory by incorporating additional managerial flexibility into firm valuations which, in turn, influences optimal financing decisions.[3] The key innovation of our model is the introduction of fixed operating costs, and hence the possibility of negative future earning due to such operating leverage. We find that in the presence of operating leverage the manager's flexibility to abandon operations in such loss-making scenarios introduces non-linearities into the project valuation that are sufficient to reconcile the trade-off theory with the observed negative earnings/financial leverage relationship. This is the main theoretical contribution of our paper. Our model also exposes the influence of the level of operating leverage on optimal financial leverage and serves managerial decision making by improving the understanding about the relationship between earnings and leverage, its effect on optimal financing, project abandonment, and optimal capital structure.

[1] The static trade-off theory predicts that more profitable firms should have more debt since expected bankruptcy costs are lower and the expected tax shield is higher.

[2] The first dynamic models to consider tax versus bankruptcy were Kane, Marcus, and MacDonald (1984) and Brennan and Schwartz (1984). Such models provide insight into both the dynamic nature of optimal leverage ratios through time and across firms.

[3] We use a real options approach which explicitly takes into account the managerial ability to revise future operating decision in response to the arrival of new information. Managers use real options as an analytical tool and as a language in which to frame investment and financing problems (Triantis and Borision, 2001). Therefore it would appear that such real optionality is important to consider from a managerial perspective.

The current paper builds on a particularly appealing dynamic trade-off model introduced in Leland (1994) which, using project value (earnings) as a lognormal stochastic state variable, provides optimal financial leverage implications; the conclusion in this lognormal setting being that the optimal leverage ratio should be *independent* of earnings. Subsequently, Sarkar and Zapatero (2003) extended Leland (1994) and demonstrated that, in this dynamic trade-off setting, the earnings/leverage relationship is dependent on the particular stochastic process assumed for the firm's earnings. Their application of a (non-negative) mean-reverting earnings process to the model of Leland (1994) provides an inverse relationship between earnings and leverage in line with empirical evidence. Furthermore, Sarkar and Zapatero (2003) state that, whilst it would be desirable, a model that allows for negative earnings would not affect their main result. On the contrary, we find that the inclusion of operating leverage (and hence possible negative earnings) turns out to contribute significantly to the *qualitative* features of the model and produces the same negative relationship without the need for the assumption of mean-reverting earnings.

The present paper can thus be seen as an extension to the models of Leland (1994) and Sarkar and Zapatero (2003). We provide valuation formulae for a wider class of earnings uncertainty processes and explicitly include fixed operating costs, allowing for the possibility of the earnings process to become negative. We also add the appropriate risk-adjustment into valuations to correctly account for the more complex risk-profiles induced by the manager's flexibility to abandon. Finally, we note that, like these previous models, we do not explicitly model agency conflicts or asymmetric information, choosing instead to consider only taxes and financial distress, the main features of the trade-off theory. Our objective is to demonstrate that the injection of operating leverage (fixed operating costs) into the dynamic trade-off setting is sufficient to explain the observed leverage/earnings relationship; even in the presence of simpler (non mean-reverting) earnings assumptions.

The rest of the paper is organised as follows: Section 2 outlines our extended model, deriving the valuation formulae and optimal financing behaviour. Section 3 investigates in detail the resulting dependence of the leverage ratio on earnings and other key model parameters. Section 4 presents empirical tests of our model's implications and Section 5 concludes.

2. The model

2.1. Assumptions

We model the representative firm as a single project which generates an uncertain income stream over time. Since our aim is to explicitly examine the effect of operating leverage on the firm's optimal financing decisions we choose to model the firm's per period earnings—interpreted as EBIT + Depreciation—as $X_t - C$, hence we split earnings into a variable *contribution margin* (sales minus variable costs), denoted X_t, and *fixed costs*, denoted $C > 0$; assumed to be constant over time. In modelling the firm's earnings this way we are able to incorporate the possibility of negative earnings—even for a positive process X.[4] In addition, we note that such modelling allows

[4]Explicitly modelling fixed costs also allows us to quantify the *degree of operating leverage* (*DOL*) of the firm. Recall from standard corporate financial theory that *DOL* is the sensitivity of changes in earnings to changes in

for uncertainty in both input and output prices since the process X is interpreted as the contribution margin, i.e. revenue minus variable costs.

To provide some generality we model $(X_t)_{t\geq 0}$ as the following diffusion process living on the filtered probability space $(\Omega, \mathrm{P}, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{F})$ and described by the SDE

$$dX_t = \mu(X_t)dt + \sigma(X_t)dW_t^{\mathrm{P}}, \quad X_0 = x, \tag{1}$$

where μ and σ are assumed to be continuous and dW_t^{P} denotes the increment of the Wiener process under the *real-world* measure P. The most commonly used stochastic process in the literature is geometric Brownian motion (GBM) since it often provides extremely tractable results. It has been argued, however, that many financial time series, including corporate earnings, may exhibit mean-reverting behaviour (not a feature of GBM). The class of diffusions in (1) encompasses many models including GBM and mean-reverting dynamics.

We also incorporate the effect of the project's risk on the firm's optimal decision making via the so-called *risk-discounting effect* highlighted by Sarkar (2003), where one must appropriately risk-adjust the discount rate used for valuations. To do so, we assume the existence of a suitable spanning asset (resulting in a complete market) and hence we apply contingent claims analysis to price the firm's real options. Such an analysis requires that expectations be taken under the equivalent *risk-neutral* measure Q and standard arguments (Dixit and Pindyck, 1994) reveal that the dynamics under this measure are given by

$$dX_t = (\mu(X_t) - \lambda\rho\sigma(X_t))dt + \sigma(X_t)dW_t^{\mathrm{Q}}, \quad X_0 = x, \tag{2}$$

where we have effectively subtracted a risk-premium ($\lambda\rho\sigma$) from the drift of the real-world price dynamics. Here λ represents the market price of risk and ρ denotes the correlation between the firm's earnings and the market (cf. Dixit and Pindyck, 1994, page 148).

The firm issues perpetual debt with a per period coupon payment of \$$R$ which remains fixed until debt is defaulted upon and hence the firm is abandoned; a time which occurs at the equityholders' discretion. We will see that it is optimal for equityholders to default on the firm when equity value is reduced to zero, a situation which occurs when the process X falls below some threshold x_d (to be determined endogenously). Upon default, we follow Leland (1994) in assuming that the bondholders receive control of the firm's assets minus a fractional bankruptcy cost (denoted by b). The presence of a tax advantage to debt financing is crucial in the trade-off model and here we denote the effective tax rate (including any adjustments for personal taxes if necessary) by τ. The tax scheme is further assumed to be symmetric in the sense that the firm will receive a tax rebate should earnings become temporarily negative. All tax benefits are assumed lost when equityholders default (Leland, 1994).

In what follows we appeal to some well known analytical probability results to derive expressions for the project and total firm values, expressions that are valid for the general class of diffusion processes given by (1).

revenue/sales and in the present setting is given by $DOL = X/(X - C)$, the contribution margin divided by EBIT + Depreciation. We see that in the absence of fixed costs $DOL = 1$ and furthermore that, as the fixed costs increase, the DOL increases, i.e. $\partial DOL/\partial C \geq 0$.

2.2. *Project value (including abandonment)*

In the absence of debt the valuation of the project is given by the expected value of the after-tax earnings stream (in present value terms). The manager of the project also has the explicit option to terminate the project, should it become optimal for them to do so. In this case the project value is given by

$$V(x) = \sup_{T_a} \mathbb{E}_x^{\mathbb{Q}} \int_0^{T_a} e^{-rt}(1-\tau)(X_t - C)dt \tag{3}$$

where r is the constant risk-free rate and T_a denotes the time for the project manager to close-down (abandon) the project. In the absence of any fixed costs (i.e. $C = 0$) it is clear that the manager will optimally keep the project running indefinitely since the earnings stream is never expected to become negative (for a positive process X). Therefore, in this case the project's expected lifespan is infinite and the optimal time to exercise the option, denoted T_a^*, will be ∞. In such a case it can be shown that, under the assumption of both GBM and the mean-reverting process of Sarkar and Zapatero (2003), the project value will be linear in the initial value of the underlying process, x. For positive fixed cost levels $C > 0$, however, the ability to abandon the project will result in a nonlinear (convex) project valuation. We will see later that this seemingly innocuous assumption will have a large impact on the qualitative relationship between earnings and optimal financial leverage.

To illustrate further the value of the abandonment option, and to aid with the solution to the optimal stopping problem, we re-write (3) in the following form (appealing to the strong Markov property of the process X)

$$V(x) = f_u(x) + \sup_{T_a} \mathbb{E}_x^{\mathbb{Q}} \left[e^{-rT_a}(-f_u(X_{T_a})) \right] \tag{4}$$

where

$$f_u(x) = \mathbb{E}_x^{\mathbb{Q}} \int_0^{\infty} e^{-rt}(1-\tau)(X_t - C)dt \tag{5}$$

represents the value of the project if it were *never* abandoned. The second term in Eq. (4) is therefore interpreted as the value of the abandonment option.

Given the infinite-horizon of the optimal stopping problem in Eq. (4) it can be shown that the optimal stopping rule is independent of time and hence takes the form of a threshold strategy, namely $T_a^* = \inf\{t \geq 0 \,|\, X_t = x_a^*\}$. As such, the optimisation over stopping times becomes an optimisation over threshold levels x_a, in other words

$$V(x) = f_u(x) + \sup_{T_a} \mathbb{E}_x^{\mathbb{Q}} \left[e^{-rT_a}(-f_u(X_{T_a})) \right] = f_u(x) + \max_{x_a} \left\{ (-f_u(x_a)) \mathbb{E}_x^{\mathbb{Q}} \left[e^{-rT_a} \right] \right\}. \tag{6}$$

Furthermore, it is well known from the theory of linear diffusions that the expected discount factor is given by

$$\mathbb{E}_x^{\mathbb{Q}} \left[e^{-rT_a} \right] = \begin{cases} \phi(x)/\phi(x_a) & \text{for } x \geq x_a, \\ \psi(x)/\psi(x_a) & \text{for } x < x_a, \end{cases} \tag{7}$$

where $\phi(x)$ and $\psi(x)$ are the unique (up to a linear scaling), positive, decreasing and increasing solutions, respectively, of the linear second-order ODE[5]

$$\frac{1}{2}\sigma^2(x)u''(x) + (\mu(x) - \lambda\rho\sigma(x))\,u'(x) - ru(x) = 0. \tag{8}$$

Putting everything together we see that the project value is given by

$$V(x) = \begin{cases} f_u(x) - f_u(x_a^*)\frac{\phi(x)}{\phi(x_a^*)}, & \text{for } x \geq x_a^*, \\ 0, & \text{for } x < x_a^*, \end{cases} \tag{9}$$

where the optimal abandonment trigger level x_a^* solves the following equation

$$\frac{\phi'(x_a^*)}{\phi(x_a^*)} = \frac{f_u'(x_a^*)}{f_u(x_a^*)}, \tag{10}$$

which is obtained from the first-order condition of the maximisation in Eq. (6).[6]

To aid with our interpretation of the results presented later we consider the risk of the project. An application of Itô's formula to (9) shows that the instantaneous variance of the unlevered project (business risk) is given by

$$\mathrm{Var}(dV/V) = \left[\frac{\sigma X_t V'(X_t)}{V(X_t)}\right]^2 dt. \tag{11}$$

For linear project valuations ($V(x) = Ax$), the case when there is no value to the abandonment option under a GBM assumption for example, it is clear that the variance of the project reduces simply to σ^2 and the risk of the project is the same as that of the underlying earnings (since C is assumed constant). More importantly, we note that the risk of the project is *independent* of X and hence the current earnings level. On the other hand, nonlinearities in the project valuation introduced by positive fixed costs C and the option to abandon results in an earnings level dependent risk. Specifically, it can be seen that the instantaneous variance in (11) is decreasing in X. Hence, as the earnings level decreases towards the abandonment trigger, the risk of the underlying project is increased considerably. This increased risk needs to be incorporated correctly into valuations and hence provides us with a strong motivation to incorporate the appropriate risk-discounting effect (cf. Sarkar, 2003).

2.3. *Firm value*

In the presence of debt the total firm value is given by the sum of debt and equity claims. The existence of a tax-shield suggests that the (levered) firm value should be greater than the (unlevered) project value.

The value of equity, E, is given by the following optimal stopping problem analogous to the value of the unlevered project

$$E(x) := \sup_{T_d} \mathbb{E}_x^{\mathbb{Q}} \int_0^{T_d} e^{-rt}(1-\tau)(X_t - C - R)dt, \tag{12}$$

[5]These functions are often called the *fundamental* solutions to such ODEs. For more details on these and the identity (7) see Chapter II, Part 11 of Borodin and Salminen (2002).

[6]The second-order condition can also be verified on a case-by-case basis.

where we recall that R denotes the debt coupon payment level. Comparison of (12) and (3) reveals that the equity value E is simply given by the unlevered project value V but with the fixed cost C replaced with $C + R$, which upon setting $f_\ell(x) = f_u(x) - (1-\tau)R/r$, yields

$$E(x) = \begin{cases} f_\ell(x) - f_\ell(x_d^*)\frac{\phi(x)}{\phi(x_d^*)}, & \text{for } x \geq x_d^*, \\ 0, & \text{for } x < x_d^*, \end{cases} \tag{13}$$

where the optimal default trigger level x_d^* solves the following equation

$$\frac{\phi'(x_d^*)}{\phi(x_d^*)} = \frac{f_\ell'(x_d^*)}{f_\ell(x_d^*)}. \tag{14}$$

To value debt we note that the periodic cash flow is equal to the coupon payment R, provided that the equityholders do not default. In the case of default, debtholders receive the value of the unlevered project less bankruptcy costs. Therefore, the debt value is given by

$$D(x) := \mathbb{E}_x^{\mathrm{Q}}\left[\int_0^{T_d^*} e^{-rt}R dt + e^{-rT_d^*}(1-b)V(X_{T_d^*})\right] \tag{15}$$

where T_d^* denotes the equityholders' optimal default time. Note that this is no longer an optimisation problem since the debtholders do not have any direct influence on the time of default. Accordingly, the debt value can be shown, using (7), to be

$$D(x) = \begin{cases} \frac{R}{r} + \left((1-b)V(x_d^*) - \frac{R}{r}\right)\frac{\phi(x)}{\phi(x_d^*)}, & \text{for } x \geq x_d^*, \\ (1-b)V(x), & \text{for } x < x_d^*. \end{cases} \tag{16}$$

We denote by FV the total firm value which is given by $FV := E + D$. Substitution of (13) and (16) into this definition and after some labourious calculations we arrive at

$$FV(x) = V(x) + \frac{\tau R}{r}\left(1 - \frac{\phi(x)}{\phi(x_d^*)}\right) - bV(x_d^*)\frac{\phi(x)}{\phi(x_d^*)}. \tag{17}$$

Therefore, we find that the value of the levered project can be expressed as the sum of three components. The value of the unlevered project, the expected additional benefit provided by debt in the form of a tax shield, and the expected cost of bankruptcy. This representation forms the basis for the trade-off theory of optimal capital structure (Baxter, 1967, Kraus and Litzenberger, 1973, Scott, 1976).

To implement the above valuations for alternative assumptions on the earnings distribution we note that it is simply a matter of determining the correct function ϕ (via Eq. (8)), computing the auxiliary functions f_u and f_ℓ, and hence solving (10) and (14) to determine the optimal abandonment and default trigger levels, x_a^* and x_d^*, respectively. In this paper we choose to illustrate the valuations using the most commonly used distributional assumption of geometric Brownian motion (GBM) in which case $\mu(x) = \mu x$ and $\sigma(x) = \sigma x$. We see that in this case $\phi(x) = x^{-\gamma}$ where $-\gamma$ is the negative solution to the quadratic $\frac{1}{2}\sigma^2\gamma(\gamma-1) + (\mu - \lambda\rho\sigma)\gamma - r = 0$. Furthermore,

$f_u(x) = (1-\tau)\left(\frac{x}{r+\lambda\rho\sigma-\mu} - \frac{C}{r}\right)$ and recall that $f_\ell(x) = f_u(x) - (1-\tau)R/r$. Using these two results we observe that in the case of GBM, equations (10) and (14) can be solved *explicitly* to obtain

$$x_a^* = KC \quad \text{and} \quad x_d^* = K(C+R), \quad \text{where} \quad K = \frac{\gamma}{1+\gamma}\left(1 - \frac{\mu - \lambda\rho\sigma}{r}\right). \tag{18}$$

Substitution of the above expressions into (9), (13), (16) and (17), provide the explicit valuation formulae under GBM dynamics.

Throughout the remainder of the paper we will illustrate the model by choosing a set of 'base case' parameters associated with the GBM assumption for the process X. The chosen parameters are: $\mu = 7\%$, $\sigma = 40\%$, $\lambda = 0.4$, $\rho = 1$, $r = 5\%$, $\tau = 15\%$, $b = 0.5$, $C = \$0.75$, and $x_0 = \$2$.

*** Insert Figure 1 about here ***

Figure 1 shows the decomposition of total firm value into its various components (for a fixed coupon R); we show the decomposition according to $FV = E + D$ in Figure 1(a) and according to (17) in Figure 1(b). We note the convexity and concavity of the equity and debt valuations, respectively, and that as the level of earnings rise the expected benefit from the tax shield rises and the expected bankruptcy costs fall—potentially providing more incentive to exploit the tax shield by taking on more fixed financing costs. We also observe that the firm value is indeed greater than that of the underlying project, but only for sufficiently high levels of current earnings in this fixed coupon example.

2.4. Optimal financing decisions

In addition to determining the optimal time at which to default on the up-and-running project, equityholders also have the flexibility to choose the level of debt employed in the firm's capital structure. Consistent with the literature, we assume that equityholders chose the coupon rate R, and hence the level of debt, in order to maximise *total firm value* since the debt is perpetual and taken on board before the firm is initiated.[7] The optimal coupon is thus defined as

$$R_* := \arg\max_R \{FV(x,R)\}.$$

We see from (17) that an increase in the coupon R increases the expected benefits of the tax shield but also increases the expected costs of bankruptcy. The trade-off of these two components clearly results in a unique optimal coupon policy. Furthermore, using (17) we see (after differentiation and some judicious manipulation) that R_* is implicitly defined by the first-order condition equation

$$\frac{\phi(x_d^*(R_*))}{\phi(x)} = 1 - BR_*\frac{\partial x_d^*}{\partial R_*}(R_*)\frac{f_\ell'(x_d^*(R_*))}{f_\ell(x_d^*(R_*))}, \tag{19}$$

where we have set $B := 1 - b(1-\tau^{-1}) \geq 1$ and we have also made clear the dependence of the optimal default trigger x_d^* on the optimal coupon R_*. Eq. (19) is particularly difficult to analyse in general since x_d^* may only be *implicitly* defined via Eq. (14). In such cases we resort to numerical investigation of the optimal coupon rate using standard root-finding algorithms applied to (19).

[7] Recall that the equityholders subsequently default in order to maximise *equity value only* (Leland, 1994).

Remark 1. We remark that in the GBM case, when x_d^* has an *explicit* solution, given by (18), Eq. (19) can be investigated further and in the zero operating leverage situation of Leland (1994), in which $C = 0$, Eq. (19) can actually be solved explicitly to obtain

$$R_* = \frac{1}{K}(1 + \gamma B)^{-\frac{1}{\gamma}} x. \tag{20}$$

When $C > 0$ we note that $x_d^* = K(C + R_*)$ but that no such explicit solution to (19) exists. We therefore resort to numerical root-finding to determine R_* in this case also.

As we will see in the following section, the dependence of the optimal coupon on the current earnings level is an important factor in determining the earning/leverage relationship. Leland (1994) finds that the optimal coupon is directly proportional to the current earnings, consistent with the finding above—Eq. (20). Sarkar and Zapatero (2003), on the other hand, find that when earnings are mean reverting the optimal coupon is very insensitive to the current earnings level.

3. The earnings / leverage relationship

We now explore the theoretical implications of our model (and the inclusion of operating leverage) on the relationship between current earnings and the optimal financial leverage ratio denoted L_* which we define as

$$L_*(x) = \frac{D(x, R_*(x))}{FV(x, R_*(x))},$$

for $x \geq x_d^*$.[8] We note that for a single firm the dependence of leverage on earnings is the same as the dependence of leverage on the contribution margin x since the fixed costs C are assumed constant.[9] We also note that, economically, as the level of the earnings (contribution margin) increases it has both a direct effect on equity and debt (and hence total firm) valuations, through higher expected total future profits etc., and a further indirect effect on the these valuations via the optimal coupon's dependence on the current earnings level. The notation used here, e.g. $D(x, R_*(x))$, makes these two dependencies clear, and in order to investigate the earnings/leverage relationship it suffices to evaluate the function $L_*(x)$. In particular, we are interested in its gradient and under what circumstances this gradient is negative.

Rather than work directly with L_* as defined above we will find it easier, and more intuitive, to use the debt-to-equity ratio, defined as $\tilde{L}_*(x) := D(x, R_*(x))/E(x, R_*(x))$. It can be easily shown (since $FV = D + E$) that

$$L_*(x) = \left(1 + \frac{E(x, R_*(x))}{D(x, R_*(x))}\right)^{-1} = (1 + \tilde{L}_*^{-1}(x))^{-1}$$

and hence the leverage ratio L_* is *decreasing* in x (and hence earnings) whenever the debt-to-equity ratio $\tilde{L}_*$ is also *decreasing* in x. Furthermore, if we wish to determine if $x \mapsto \tilde{L}_*(x)$ is decreasing

[8] Note that, trivially, the leverage below x_d^* is equal to one since the firm has no equity.

[9] When comparing different firms however, as we will do empirically in Section 4, we note that the earnings/leverage relationship will also depend on the relationship between the level of fixed costs and optimal leverage.

we can differentiate fully with respect to x to obtain

$$\frac{\tilde{L}'_*(x)}{\tilde{L}_*(x)} = \underbrace{\left(\frac{1}{D}\frac{\partial D}{\partial x} - \frac{1}{E}\frac{\partial E}{\partial x}\right)}_{\text{Valuation Effect}} + \underbrace{\left(\frac{1}{D}\frac{\partial D}{\partial R_*} - \frac{1}{E}\frac{\partial E}{\partial R_*}\right) R'_*(x)}_{\text{Financing Effect}} . \tag{21}$$

The two components of this full derivative provide us with a tool to understand the total effect of earnings on optimal leverage. The first component, quantified by the first term in brackets, represents the *relative* effect of the valuations of both debt and equity, for a *fixed* coupon R_*, as the level of earnings change. It is clear that the value of both debt and equity increase as the current earnings increase,[10] however the relative sizes of the increases (i.e. elasticity) is the important factor in determining the first contribution to the debt-to-equity ratio. It can be shown that the first term is negative since the elasticity of the equity value to current earnings is higher than that of debt; hence for a \$1 change in earnings the equity value changes by more than the debt value (in percentage terms).[11] We therefore conclude that, in the absence of the indirect effect of current earnings on the optimal coupon (the second term above), the debt-to-equity ratio would be decreasing in x, and hence current earnings.

In general, however, as earnings change it may become optimal for the firm to have a different coupon level R_* in order to optimise total firm value and so this additional effect on the optimal coupon R_* will contributes to the overall dependence of leverage on current earnings. This is the effect quantified by the second component on the right hand side in Eq. (21). We note that differing coupon levels influence the value of equity and debt in two ways: the first is via a change in the actual cashflows to the equityholders and debtholders, and the second is via its effect on the optimal default strategy of the equityholder. Both effects need to be taken into account when determining the effect earnings level has on optimal leverage. It can be seen that the sign of the second term in brackets in (21) is positive since, ceteris paribus, equity is a decreasing function, and debt an increasing function, of the coupon level R_*. We therefore would conclude that the debt-to-equity ratio $\tilde{L}_*$ is decreasing in x if $R'_* \leq 0$. Unfortunately, this is not the case—in fact we will see below that R_* is increasing in x—and as such the overall effect of earnings on leverage remains analytically ambiguous; hence we resort to numerical investigations (see Section 3.1).

Expression (21) does, however, provide additional insights into existing results from the literature. For the mean-reverting case of Sarkar and Zapatero (2003), these authors demonstrate that the long-run level of earning (a parameter in the mean-reverting process) dominates financing decisions and so the optimal coupon is extremely insensitive to the current level of earnings, hence $R'_* \approx 0$. In this case it is clear from (21) that an inverse relationship between current earnings and optimal leverage will ensue. Furthermore, we also observe that in the GBM setting of Leland

[10]We note further that a change in earnings affects the valuation of debt and equity in subtlety different ways. For equity it is clear that a higher current level of earnings results in a higher present value of total future earnings (all else fixed) and hence contributes to a higher valuation. However, since the level of the cash-flow stream to debt is fixed, debt valuation is only affected indirectly by the current level of earnings in so far as current earnings affect the expected time until the equityholders default on the levered project. A higher level of current earnings prolongs the expected lifespan of the project and hence the present value of the total cash-flow stream to debt is also increased.

[11]For a proof see Appendix A.

(1994) the magnitude of the second term in (21) *exactly* balances that of the first, resulting in a constant debt-to-equity (and hence leverage) ratio. This finding should be seen as a consequence of a fortuitous interaction of a linear valuation of the underlying project and the particular assumption of the earnings process (GBM), rather than reflecting economic reality. In general, the two terms in (21) will not balance exactly. We will see below that, for the simple GBM case, the inclusion of fixed operating costs results in a positive relationship between the optimal coupon R_* and current earnings but that this relationship is nonlinear (and in fact concave), i.e. $R'_* \geq 0$ and $R''_* \neq 0$.

To investigate the sign of R'_* more closely we differentiate (19) implicitly with respect to x—noting the dependence of x^*_d on R_* which is itself dependent on x. The sign of the resulting expression cannot be determined in general but for the case of GBM we observe that the aforementioned differentiation yields

$$R'_*(x) = \frac{K^{-\gamma}\left(\frac{C+R_*}{x}\right)^{1-\gamma}}{1 + B\left[\frac{C+\gamma R_*}{C+R_*}\right]}, \tag{22}$$

which can be seen to be positive for all parameter values. Hence, even in the presence of operating leverage, it is optimal to take on a higher coupon payment as the current level of x, and hence earnings, increase.

Remark 2. To check for consistency we observe that for $C = 0$ Eq. (22) reduces to the ODE $R'_*(x) = \frac{K^{-\gamma}}{1+\gamma B}\left(\frac{R_*}{x}\right)^{1-\gamma}$ which admits the linear solution $R_*(x) = K^{-1}[1+\gamma B]^{-\frac{1}{\gamma}}x$—corroborating Eq. (20).

3.1. Numerical Results

This section illustrates our model results fully in the case of GBM for $C \neq 0$ and points briefly to further results from the mean-reverting world. In particular, we demonstrate numerically that the derivative in Eq. (21) is indeed negative for $C \geq 0$ and hence a negative relationship exists between earning and financial leverage in the presence of fixed costs. Note that our numerical results where repeated for the entire range of parameter values, finding this result to be robust to changes in the base case parameters.

*** Insert Figure 2 about here ***

Figure 2(a) shows the dependency of the optimal coupon on the current level of x (contribution margin) for varying degrees of fixed costs C (operating leverage). As expected from Eq. (20) we observe that, in the absence of operating leverage, the optimal coupon is linear. Furthermore, we see that the inclusion of operating leverage produces an increasing concave relationship and that the optimal coupon is in fact zero when the current level of x is equal to the abandonment trigger x^*_a. This result is perfectly intuitive, since when earnings are so low that it is optimal for the underlying project to be abandoned there is no value in the project and taking on any debt, however small, will produce negative equity values.

Figure 2(b) shows the relationship between operating leverage (C) and the optimal coupon, for various (fixed) levels of the current contribution margin x. We note that since the contribution margin is fixed an increasing C would result in a decrease in corporate earnings. Perhaps counterintuitively, we observe that the relationship between C and R_* is in fact non-monotonic, with the optimal coupon actually increasing initially as the degree of operating leverage increases and hence earnings decrease. A maximum optimal coupon is attained and then the optimal coupon declines as the fixed costs become too high and earnings too low. This result highlights the delicate relationship between operating leverage and optimal financing decisions. The source of this non-monotonicity is left for future research as it is not the focus of the present study, however the explanation no doubt lies in the trade-off between the tax-shield and the costs of bankruptcy.

*** Insert Figure 3 about here ***

Next, Figure 3 shows our main theoretical result, the dependency of the optimal leverage on current earnings level, when its affect on the optimal coupon is correctly taken into account. We can see clearly, from Figure 3(a), the negative relationship between the contribution margin x and the optimal leverage when fixed costs are added to the underlying project ($C > 0$). When such operating leverage is removed, the optimal leverage becomes independent of x (and hence earnings) consistent with Leland (1994). In addition, we show in Figure 3(b) the relationship between the optimal financial leverage and the level of the fixed costs for various (fixed) current levels of the contribution margin. We see that, despite the non-monotonic behaviour of the optimal coupon rate in the level of C, the relationship between optimal financial leverage and C is monotonic and increasing. This is due to the additional effects of operating leverage on project risk, equityholders' optimal timing, and therefore the market valuations of both debt and equity. These two observations lead us to highlight the following result:

Result 1. *In the presence of non-zero fixed costs the optimal financial leverage is a decreasing function of the contribution margin and an increasing function of operating costs. Hence there is a negative relationship between the optimal financial leverage and the current level of earnings (irrespective of whether earnings change due to a change in the contribution margin or fixed costs).*

We also observe from Figure 3(b) that for a given level of the contribution margin x there exists a maximum level of operating leverage (fixed cost C) which the project can bear. At this maximum level, the underlying project has a valuation of zero and should be abandoned at the current level of x. Intuitively, this capacity for operating leverage is increased when the level of the current contribution margin is higher.

Figure 3(b) also provides the following result in regards to the shape of the relationship between financial leverage and both the contribution margin x and fixed costs C:

Result 2. *The optimal financial leverage L_* is a* convex *function of the contribution margin x and a* concave *function of the level of the fixed costs C.*

Finally, while we do not present the results here in the interests of brevity, we note that the inclusion of a non-zero fixed costs ($C > 0$) into the mean-reverting earnings model of Sarkar and

Zapatero (2003) provides consistent results to those described above.[12] Since mean reversion and operating leverage both result in a decrease of optimal financial leverage as earnings increase this relationship is maintained in the presence of the two effects. It can be seen, however, that for levels of the fixed costs sufficient to introduce the possibility of abandonment, the insensitivity of the optimal coupon R_* to changes in current earnings (Sarkar and Zapatero, 2003) is no longer observed. In the presence of abandonment, the additional risks introduced into the valuation lead to significant earnings dependency in the optimal coupon, indicating that the long-run earnings level is no longer the dominant factor in the mean-reverting firm's optimal financing decisions.[13]

3.2. Empirical implications

Our model renders a number of empirical predictions regarding the relationship between fixed costs (operating leverage), corporate contribution margins and earnings, and financial leverage. The model predicts an inverse (convex) relationship between the current level of the contribution margin and a firm's financial leverage. It also suggests that a firm's financial leverage would be higher for those companies with a higher level of fixed cost, but that this relationship is a concave one.[14] Whilst these implications are drawn from a model of a single firm, it is clear that the cross-sectional effects of the model parameters on financial leverage would also exhibit the same relationships and hence our model implications are tested in the cross-section.

4. Empirical Findings

This section provides some empirical evidence in support of our model implications. There have been numerous cross-sectional tests of capital structure theories examining whether leverage ratios vary across firms as the theory predicts. The literature is extensive and we do not attempt to reproduce it here, instead we focus on the new implications of our model in regards to the influence of the contribution margin and fixed costs on a firm's financial leverage.[15] We point the interested reader to Frank and Goyal (2008)—Section 3.2 in particular—and the references therein for details of related empirical studies.

Our empirical analysis consists of three empirical model specifications regressing a firm's market leverage (LEVERAGE) on various proxies for our model parameters. Since our theoretical model relates to the market valuation of both debt and equity, our model implications are valid for the *market* leverage ratio. Recent literature has noted the importance of such valuation components and empirical attention has shifted towards explanations of market leverage (as opposed to book leverage) in recent years. For example, Welch (2004) argues that market equity is much preferable

[12]Results are available from the authors upon request.

[13]Furthermore, it was observed that for certain parameter regimes under both mean-reversion and operating leverage it was possible to obtain a U-shaped leverage pattern in current earnings. This supports the difficulties discussed above in proving the decreasing relationship in full generality. Such non-linearity is left for the subject of future research.

[14]In full, the model predicts that financial leverage is an increasing function of the discount parameters (λ, ρ, and r), the tax rate (τ), and fixed costs (C), and a decreasing function of the process parameters (μ and σ), bankruptcy costs (b), and the contribution margin (x).

[15]To our knowledge there is no existing empirical analysis of the effect of fixed costs on leverage in a trade-off theory context.

over book equity since book equity is merely a 'plug figure'. However, for a large number of firms the market value of debt is not available since the debt instruments employed are not publicly traded. We therefore proxy the market leverage as the book value of long-term-debt dividend by sum of the book value of long-term-debt and the market value of equity (cf. Titman and Wessels, 1988).

Our regressors include DRIFT and SIGMA, which are parameter estimations of the drift and volatility terms, μ and σ, describing the GBM dynamics of a firm's contribution margin. We also control for each firm's effective corporate tax rate (TAXRATE) and we proxy the correlation ρ used for risk-adjustment with the regressor BETA, an estimate of the firm's market risk. We also choose return-on-assets, ROA [=(EBIT+Depreciation)/Total Assets], to represent a normalised earnings measure for the cross-sectional comparison of firms. To proxy a firm's fixed costs we choose SG&A (Sales, General & Admin), again normalised by total assets.[16] Finally, CMARGIN is the firm's contribution margin normalised by total assets. Similar to Sarkar and Zapatero (2003) we note that there are no suitable proxies for the bankruptcy cost b and that there is no variation in r or λ across firms. We therefore omit these variables from our empirical tests.

As a benchmark we first perform the regression[17]

$$\text{LEVERAGE}_{i,t} = \beta_0 + \beta_1 \text{DRIFT}_{i,t} + \beta_2 \text{SIGMA}_{i,t} + \beta_3 \text{TAXRATE}_{i,t} + \beta_4 \text{BETA}_{i,t} + \beta_5 \text{ROA}_{i,t} + \epsilon_{i,t} \quad (23)$$

to determine whether or not the well documented negative relationship between earnings and leverage is present in our dataset. Our main empirical predictions are tested using two model specifications. The first builds on the benchmark above and is given by

$$\begin{aligned} \text{LEVERAGE}_{i,t} = {} & \beta_0 + \beta_1 \text{DRIFT}_{i,t} + \beta_2 \text{SIGMA}_{i,t} + \beta_3 \text{TAXRATE}_{i,t} \\ & + \beta_4 \text{BETA}_{i,t} + \beta_{51} \text{CMARGIN}_{i,t} + \beta_{52} \text{SG\&A}_{i,t} + \epsilon_{i,t} \end{aligned} \quad (24)$$

where we have effectively split earnings into the contribution margin and SG&A, analogous to our theoretical contribution. Note that following the theoretical model predictions we expect β_1, β_2, β_5, and β_{51} to be negative and β_3, β_4, and β_{52} to be positive.

Finally, we test our model's implications in regards to the nonlinearity of the relationships between leverage and the contribution margin and fixed costs by performing the further augmented regression

$$\begin{aligned} \text{LEVERAGE}_{i,t} = {} & \beta_0 + \beta_1 \text{DRIFT}_{i,t} + \beta_2 \text{SIGMA}_{i,t} + \beta_3 \text{TAXRATE}_{i,t} + \beta_4 \text{BETA}_{i,t} + \beta_{51} \text{CMARGIN}_{i,t} \\ & + \beta_{52} \text{SG\&A}_{i,t} + \beta_{61} \text{CMARGINsq}_{i,t} + \beta_{62} \text{SG\&Asq}_{i,t} + \epsilon_{i,t} \end{aligned} \quad (25)$$

where CMARGINsq and SG&Asq denote the square of the variables CMARGIN and SG&A, respectively. The theoretical model predicts LEVERAGE to have a convex relationship with CMARGIN and a concave relationship with SG&A and hence we expect $\beta_{61} > 0$ and $\beta_{62} < 0$.

In the following we discuss sample selection and provide descriptive statistics of our sample. We then provide empirical analysis of the benchmark model (23) and the two specifications of our main model (24) and (25), coupled with the necessary robustness checks.

[16] Hence we are implicitly assuming that such costs are inelastic to changes in earnings.

[17] Note that the subscripts refer to firm i and year t, respectively.

4.1. Data, variables, and descriptive statistics

From COMPUSTAT we obtain quarterly financial data for firms in the S&P 500 from 1996 to 2011 to estimate the GBM parameters of the contribution margin process. This provides an initial sample of 48,899 quarter observations of 968 S&P 500 firms. Following Sarkar and Zapatero (2003) our process parameter estimations for each year from 2006–2011 are based on 48 quarters whereby firms are excluded with less than 40 consecutive observations; resulting in a final sample of 43,760 quarter observations of 718 S&P 500 firms between 1996 and 2011.[18]

Annual financial reporting data for S&P 500 companies between 2006 and 2011 are also collected from COMPUSTAT, with matching firm level tax data provided by Professor John Graham.[19] Similar to Sarkar and Zapatero (2003) we conduct cross-sectional regressions for two consecutive periods using firms that are listed in the S&P 500 index in both years; two-year periods allow for consistent sub-samples without sacrificing observations over several years. In particular, our analysis focuses on three sub-periods which are pre-global financial crisis (2006-2007), financial crisis (2008-2009), and post-financial crisis (2010-2011). These periods are chosen to assess the robustness of our theoretical model predictions before, during, and after the crisis.

After removal of missing observations this results in annual observations ranging from 213 to 271 between 2006 and 2011. We exclude missing values in either year of a two-year sub-period which leaves us with final sample sizes of 200 (2006–2007), 248 (2008–2009) and 258 (2010–2011) firms.

*** Insert Table 1 about here ***

Table 1 shows the summary statistics for all firms used in at least one of the three two-year periods; 293 firms in total.

4.2. Results, robustness checks, and discussion

Table 2 reports the estimation results of Eq. (23). Importantly, and as expected, the relationship between earnings (ROA) and leverage is statistically significant and negative with the estimated coefficient being highly significant at the 1% level in every year. The explanatory power observed ranges from 27% to 43%, consistent with the findings of Frank and Goyal (2009) who find that the (six) factors significant in explaining leverage ratios account for approximately 30% of cross-sectional variation.

*** Insert Table 2 about here ***

[18] We note that the estimation procedure is standard due to the normality of the contribution margin log-differences under the assumption of GBM dynamics. Further details are omitted in the interests of brevity but are available from the authors upon request.

[19] Available via `https://faculty.fuqua.duke.edu/~jgraham/taxform.html`.

*** Insert Table 3 about here ***

Table 3 shows the results of splitting earnings into the contribution margin and fixed costs, as described by Eq. (24). As expected, CMARGIN coefficients are negative and SG&A coefficients are positive, all significant at the 1% level with the exception of a 5% significance for SG&A in 2006 and a 10% significance in 2009.[20]

The results for the remaining model parameters used as control variables are somewhat mixed. There is a consistent negative relationship between the growth of the contribution margin (DRIFT) and leverage as predicted, with significance in all years aside from the most recent 2010–2011 period—indicating possible after-effects of the financial crisis on this relationship.[21] In contrast, the SIGMA coefficient is not significant in any year, indicating a weak relationship between earnings volatility and leverage. This finding is consistent, however, with previous empirical studies that have often found the effect of earnings volatility to generally be insignificant (Titman and Wessels, 1988).

The effect of the tax rate is also found to be weak and insignificant in all but one year, indicating that variations in tax rates across firms do not appear to be a strong driver of leverage. However, the unimportance of the *existence* of a tax shield in determining leverage should not be inferred from this result. Finally, the effect of BETA on leverage is found to be significantly positive (in line with predictions) in three out of six years. Furthermore, BETA is highly significant (to the 1% level) during the period of the financial crisis (2007–2008), indicating that risk-adjustment was perhaps an important consideration of leverage decisions during this period.

Overall, Table 3 provides strong support for our theoretical model's predictions of the impact of fixed costs on observed optimal leverage ratios.

*** Insert Table 4 about here ***

Finally, we consider the predicted nonlinearities in the dependence of leverage on CMARGIN and SG&A. Table 4 shows the results of regression (25). Providing further support to our theoretical predictions, we observe very strong evidence of convexity in the contribution margin relationship; significant to at least the 10% level in all years and to the 1% level in four out of six years. Additionally, we observe strong evidence of concavity in the fixed cost relationship, more so in recent years (2009 onwards).

[20] As a robustness check of this key result, and to address the potential problem of multicollinearity of the two key regressors CMARGIN and SG&A, we orthogonalised these two variables and re-estimated Eq. (24). Results are consistent, providing identical signs and similar statistical significance, indicating that our results are not driven by the relationship between CMARGIN and SG&A. Furthermore, in another unreported regression we also controlled for industry dummies, finding that the signs and significance of CMARGIN and SG&A remained unchanged. Further details on these robustness checks are available from the authors upon request.

[21] It is postulated that the loss in significance is a consequence of DRIFT being a historical measure of a firm's growth which has been temporarily deflated by the financial crisis and does not, therefore, correctly reflect manager's growth forecasts.

5. Conclusions

In the context of the trade-off theory of capital structure, existing theoretical modelling provides contradictory conclusions on the relationship between corporate earnings and optimal financial leverage. However, strong empirical evidence of an inverse relationship between these two quantities calls for a reconciliation of theory and evidence. Previous studies have used mean-reverting earnings to provide such a reconciliation, however the present paper relaxes such strong assumptions and shows that the relationship need not be process specific and can, instead, be a direct result of correctly incorporating the effect of fixed operating costs (operating leverage), and hence the possibility of *non-financing* related distress, into project and firm valuations.

Our theoretical model also provides insights into the relationship between fixed costs and financial leverage, postulating a positive relationship between these two quantities. Empirical analysis employing data from various cross-sections of S&P 500 firms confirms this relationship and other theoretical predictions made by our model. Importantly, many results hold true for periods before, during, and after the global financial crisis, indicating robustness of our empirical findings.

Finally, we note that since operating revenue and cost structures depend heavily on the industry in which a firm is situated, and noting the empirical finding that industry leverage has been found to be a strong predictor of firm leverage (Frank and Goyal, 2008), our results in regards to the effect of fixed costs on leverage may yet provide further insights into the effect of industry membership on financial leverage. This is currently an avenue of ongoing research.

6. Acknowledgements

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Appendix A. Proof that $\frac{1}{D}\frac{\partial D}{\partial x} - \frac{1}{E}\frac{\partial E}{\partial x} \leq 0$

Proof. To demonstrate that $\frac{1}{D}\frac{\partial D}{\partial x} - \frac{1}{E}\frac{\partial E}{\partial x} \leq 0$ is the same as demonstrating that $D\frac{\partial E}{\partial x} - E\frac{\partial D}{\partial x} \geq 0$ for a fixed coupon R. We note that after substitution of (13) and (16) we have

$$\begin{aligned}\left(D\frac{\partial E}{\partial x} - E\frac{\partial D}{\partial x}\right)(x) &= \phi(x_d^*)\Big((R/r) f_\ell'(x)[\phi(x_d^*) - \phi(x)] - (R/r)\phi'(x)[f_\ell(x_d^*) - f_\ell(x)] \\ &\qquad + (1-b)V(x_d^*)[f_\ell'(x)\phi(x) - f_\ell(x)\phi'(x)]\Big) \\ &= \phi(x_d^*)\Big((R/r)(x - x_d^*)[\phi'(x) f_\ell'(\xi_1) - \phi'(\xi_2) f_\ell'(x)] \\ &\qquad + (1-b)V(x_d^*)[f_\ell'(x)\phi(x) - f_\ell(x)\phi'(x)]\Big)\end{aligned}$$

where we have invoked the *mean value theorem* to rewrite $g(x) - g(x_d^*) = g'(\xi)(x - x_d^*)$ for some $\xi \in (x_d^*, x)$. To proceed further we note that if the function f_ℓ is linear in x then $f_\ell'(\xi) = f_\ell'(x)$ for all ξ, hence the above becomes

$$\left(D\frac{\partial E}{\partial x} - E\frac{\partial D}{\partial x}\right)(x) = \phi(x_d^*)\Big((R/r)(x - x_d^*) f_\ell'(\xi_1)[\phi'(x) - \phi'(\xi_2)] + (1-b)V(x_d^*)[f_\ell'(x)\phi(x) - f_\ell(x)\phi'(x)]\Big).$$

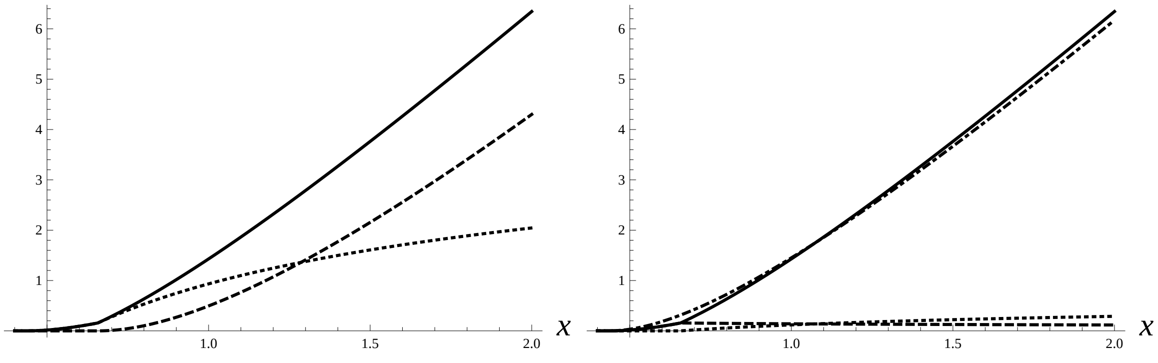
We can furthermore see that if ϕ' is increasing (hence ϕ is convex) then $\phi'(x) - \phi'(\xi_2) \geq 0$ and hence the first term is positive.

To prove the positivity of the second term we note that when $x = x_d^*$ this term is zero by (14). We can furthermore demonstrate that

$$(f_\ell'(x)\phi(x) - f_\ell(x)\phi'(x))' = -f_\ell(x)\phi''(x)$$

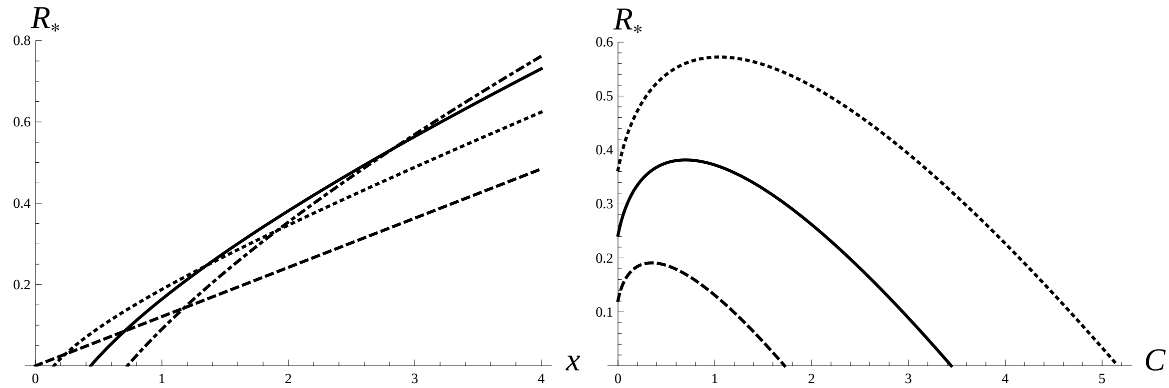
which, by our assumption of the convexity of ϕ, indicates that the function $f_\ell'\phi - f_\ell\phi'$ is increasing on the interval $x \in (x_d^*, f_\ell^{-1}(0))$ and decreasing on $x \in (f_\ell^{-1}(0), \infty)$. Given these facts and the uniqueness of the solution x_d^* to (14)—which was proved in Glover and Hambusch (2012)—we therefore conclude that the second term must remain positive on the interval $x \in (x_d^*, \infty)$, and the proof is complete. $\square$

Figures



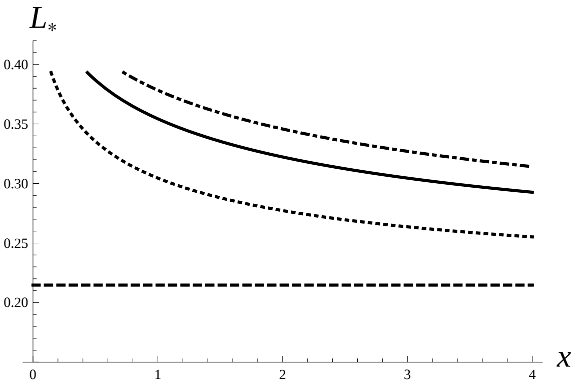
(a) $FV = E + D$ (b) $FV = V + TS - BC$

Figure 1: The decomposition of total firm value (solid line). Figure (a) shows the decomposition into equity (dashed line) and debt (dotted line) and Figure (b) shows the decomposition into unlevered project value (dot-dashed line), expected benefit from the tax shield (dotted line), and the expected bankruptcy costs (dashed line). *NB*: base case parameters are $\mu = 7\%$, $\sigma = 40\%$, $\lambda = 0.4$, $\rho = 1$, $r = 5\%$, $\tau = 15\%$, $b = 0.5$, $C = \$0.75$, and $x_0 = \$2$. The value of the coupon R used is the optimal coupon rate for the base case parameters, found to be $R_* = \$0.38$.

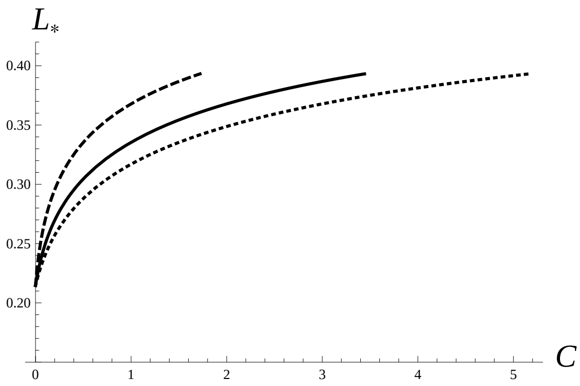


(a) Optimal coupon vs. contribution margin (b) Optimal coupon vs. fixed costs

Figure 2: On the left: The dependency of the optimal coupon on the current level of x (contribution margin) for varying degrees of operating leverage—$C = \$0.75$ (solid line), $C = \$ = 0.25$ (dotted line), $C = \$1.25$ (dot-dashed line), and $C = \$0$ (dashed line). On the right: The relationship between the optimal coupon and the degree of operating leverage C for varying levels of the contribution margin—$x = \$1$ (dashed line), $x = \$2$ (solid line), and $x = \$3$ (dotted line); for base case parameters. *NB*: base case parameters are $\mu = 7\%$, $\sigma = 40\%$, $\lambda = 0.4$, $\rho = 1$, $r = 5\%$, $\tau = 15\%$, $b = 0.5$, $C = \$0.75$, and $x_0 = \$2$.



(a) Financial leverage vs. contribution margin



(b) Financial leverage vs. fixed costs

Figure 3: On the left: The dependency of the optimal leverage on the current level of x (contribution margin) for varying degrees of operating leverage—C = \$0.75 (solid line), C = \$ = 0.25 (dotted line), C = \$1.25 (dot-dashed line), and C = \$0 (dashed line). On the right: The relationship between the optimal leverage and the degree of operating leverage C for varying levels of the contribution margin—x = \$1 (dashed line), x = \$2 (solid line), and x = \$3 (dotted line); for base case parameters. *NB*: base case parameters are $\mu = 7\%$, $\sigma = 40\%$, $\lambda = 0.4$, $\rho = 1$, $r = 5\%$, $\tau = 15\%$, $b = 0.5$, C = \$0.75, and x_0 = \$2.

Tables

Table 1: Summary statistics for the cross-section of all firms ($N = 293$) used in at least one of the three two-year periods.

Variable	Median	Mean	Std. dev.	Min	Max	Skewness	Kurtosis
LEVERAGE	0.151	0.195	0.168	0.000	0.943	1.300	4.606
DRIFT	0.100	0.144	0.196	–0.095	2.967	6.367	70.871
SIGMA	0.148	0.213	0.223	0.020	2.409	3.806	24.854
BETA	1.015	1.072	0.481	0.029	3.341	0.803	4.059
TAXRATE	0.350	0.334	0.064	0.000	0.391	–3.655	15.683
ROA	0.153	0.159	0.079	–0.112	0.625	0.635	5.096
CMARGIN	0.334	0.368	0.207	0.000	1.411	1.065	4.960
CMARGINsq	0.111	0.179	0.211	0.000	1.992	3.242	19.601
SG&A	0.178	0.210	0.168	0.002	1.094	1.508	6.406
SG&Asq	0.032	0.072	0.123	0.000	1.196	4.353	29.418

Variables: LEVERAGE = market leverage calculated as the book value of long-term-debt dividend by sum of the book value of long-term-debt and the market value of equity.
DRIFT = estimated drift rate of the contribution margin time-series under the assumption of GBM dynamics.
SIGMA = estimated volatility (standard deviation) the contribution margin time-series under GBM dynamics.
TAXRATE = pre-interest tax rate (courtesy of Professor John Graham).
BETA = estimated (S&P 500) market β of the firm (courtesy of COMPUSTAT).
ROA = return-on-assets, calculated as (EBIT+depreciation)/total assets, our normalised measured of earnings.
CMARGIN (CMARGINsq) = the contribution margin (squared) normalised by total assets.
SG&A (SG&Asq) = the sales, general and admin (squared) normalised by total assets.

Table 2: Regression results—Eq. (23). Dependent variable = LEVERAGE, t-stats in brackets, and *,**,*** represent statistical significance at the 10%, 5%, and 1% level, respectively, based on robust (White) standard errors.

		Year					
Variable	**Exp. Sign**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**
CONST.		0.3719***	0.3176***	0.1971**	0.1960***	0.3584***	0.2673***
		(4.12)	(3.95)	(2.20)	(3.02)	(4.02)	(4.35)
DRIFT	–	–0.2813***	–0.2981***	–0.3628***	–0.2358**	–0.1079	–0.0876
		(–2.92)	(–2.78)	(–3.09)	(–2.21)	(–1.54)	(–1.15)
SIGMA	–	0.2153**	0.1995**	0.1535*	0.1213	0.0739	0.071
		(2.23)	(2.05)	(1.73)	(1.39)	(1.13)	(0.95)
BETA	+	–0.0096	0.0349	0.1759***	0.0741***	0.0163	0.0512**
		(–0.56)	(1.62)	(6.15)	(3.83)	(0.63)	(2.24)
TAXRATE	+	–0.3604	–0.1062	0.1838	0.0873	–0.1608	0.0989
		(–1.43)	(–0.49)	(0.93)	(0.59)	(–0.71)	(0.69)
ROA	+	–0.5790***	–0.8556***	–1.0411***	–0.7068***	–0.8957***	–0.9873***
		(–4.82)	(–5.38)	(–7.85)	(–4.17)	(–7.41)	(–6.66)
# obs		200	200	248	248	258	258
R^2		27.25%	29.93%	43.49%	30.39%	27.39%	26.75%
Adj-R^2		25.37%	28.12%	42.33%	28.95%	25.95%	25.30%
F-stat		7.439	9.802	37.28	20.49	18.19	16.33
p-val		< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001

Table 3: Regression results—Eq. (24). Dependent variable = LEVERAGE, t-stats in brackets, and *,**,*** represent statistical significance at the 10%, 5%, and 1% level, respectively, based on robust (White) standard errors.

		Year					
Variable	**Exp. Sign**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**
CONST.		0.4378***	0.3417***	0.2564***	0.2419***	0.4003***	0.2907***
		(5.04)	(4.18)	(3.01)	(4.00)	(4.50)	(4.39)
DRIFT	–	–0.2461***	–0.2768**	–0.3324***	–0.2070**	–0.0877	–0.0771
		(–2.70)	(–2.58)	(–2.81)	(–1.98)	(–1.18)	(–0.98)
SIGMA	–	0.1438	0.1577	0.0959	0.0668	0.0403	0.051
		(1.55)	(1.57)	(1.07)	(0.76)	(0.57)	(0.64)
BETA	+	–0.0195	0.0355	0.1649***	0.0738***	0.0075	0.0466**
		(–1.15)	(1.63)	(5.89)	(4.11)	(0.29)	(2.00)
TAXRATE	+	–0.4224*	–0.1339	0.1425	0.0564	–0.2018	0.073
		(–1.79)	(–0.61)	(0.76)	(0.40)	(–0.91)	(0.50)
CMARGIN	–	–0.5006***	–0.8027***	–0.9703***	–0.5829***	–0.8204***	–0.9351***
		(–4.45)	(–4.85)	(–7.25)	(–3.53)	(–6.20)	(–5.83)
SG&A	+	0.3274**	0.7173***	0.7978***	0.3679*	0.6851***	0.8583***
		(2.50)	(3.70)	(4.88)	(1.80)	(3.84)	(4.20)
# obs		200	200	248	248	258	258
R^2		31.78%	30.83%	45.22%	33.67%	28.87%	27.19%
Adj-R^2		29.66%	28.68%	43.86%	32.02%	27.17%	25.45%
F-stat		9.512	9.786	35.94	20.96	19.49	15.41
p-val		< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001

Table 4: Regression results—Eq. (25). Dependent variable = LEVERAGE, t-stats in brackets, and *,**,*** represent statistical significance at the 10%, 5%, and 1% level, respectively, based on robust (White) standard errors.

		Year					
Variable	**Exp. Sign**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**
CONST.		0.5199***	0.4485***	0.3514***	0.3945***	0.4962***	0.3875***
		(5.20)	(4.72)	(3.88)	(5.63)	(5.43)	(5.13)
DRIFT	–	–0.2213**	–0.2419**	–0.3189***	–0.1453	–0.0838	–0.0738
		(–2.55)	(–2.34)	(–2.67)	(–1.40)	(–1.21)	(–0.98)
SIGMA	–	0.1027	0.1117	0.0652	-0.0169	0.0131	0.0218
		(1.09)	(1.11)	(0.71)	(-0.19)	(0.19)	(0.27)
BETA	+	–0.018	0.0396*	0.1519***	0.0615***	0.0064	0.0512**
		(–1.01)	(1.84)	(5.15)	(3.47)	(0.24)	(2.11)
TAXRATE	+	–0.4151*	–0.133	0.1008	0.0021	–0.2122	0.0688
		(–1.87)	(–0.56)	(0.55)	(0.01)	(–0.96)	(0.50)
CMARGIN	–	–0.9655***	–1.5997***	–1.2909***	–1.4442***	–1.4883***	–1.7314***
		(–3.51)	(–6.22)	(–6.21)	(–5.43)	(–7.27)	(–7.45)
CMARGINsq	+	0.5160**	0.8362***	0.4323*	1.1188***	0.9089***	1.0233***
		(2.06)	(3.62)	(1.85)	(4.09)	(3.56)	(3.55)
SG&A	+	0.4335*	1.2135***	0.7409***	0.6491**	0.9341***	1.2897***
		(1.92)	(5.10)	(3.31)	(2.24)	(4.07)	(5.01)
SG&Asq	–	–0.308	–0.8859***	–0.1418	–0.7282**	–0.7639**	–1.0454**
		(–1.06)	(–3.25)	(–0.45)	(–2.17)	(–2.37)	(–2.50)
# obs		200	200	248	248	258	258
R^2		34.91%	35.43%	46.87%	37.99%	32.54%	30.82%
Adj-R^2		32.18%	32.73%	45.09%	35.92%	30.37%	28.59%
F-stat		11.46	11.37	28.75	19.68	18.12	15.54
p-val		< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001