QUANTITATIVE FINANCE
RESEARCH CENTRE

UNIVERSITY OF
TECHNOLOGY SYDNEY

QUANTITATIVE FINANCE RESEARCH CENTRE

Research Paper 332 May 2013

Does More Frequent Trading Increase the Volatility? – Theoretical Evidence at Asset and Portfolio Level

KiHoon Jimmy Hong

ISSN 1441-8010 www.qfrc.uts.edu.au

Does More Frequent Trading Increase the Volatility? – Theoretical Evidence at Asset and Portfolio Level

KiHoon Jimmy Hong[a]

Current Version: May 22, 2013

[a] *University Technology of Sydney*
Tel: 61 2 9514 4028 E-mail: hong.jimmy@gmail.com
Address: Cnr Quay Street and Ultimo Road, Haymarket, NSW 2000, Australia

Abstract

This paper investigates the sensitivity of asset and portfolio price volatility with respect to the minimum available trading interval that the price is quoted. The objective of the study is to find the theoretical impact of high frequency trading on asset and portfolio volatilities, using a simple stochastic model. The paper finds that if high frequency trading is available, both asset and portfolio price volatility tend to decrease. The result suggests that the regulators who are concerned with the volatility induced by high frequency trading should concentrate the regulatory effort on the behavioral aspect of the high frequency traders rather than on how frequent they trade.

JEL classification: G10, G18

Keywords: High Frequency Trading, Volatility, Technical Analysis, Time Series Momentum

1. Introduction

High frequency trading (HFT hereafter) is generally believed to lower the cost of trading, increase the informativeness of quotes, provide liquidity and eliminate arbitrage (See Hendershott, Jones and Menkveld, 2010) through fast and more frequent trading. The benefits of HFT seem to be clear and generally agreed. However there is a controversy over its cost – rise in price volatility. Traders often claim that HFT reduces the stock market volatility. The critiques of HTF do not agree with this. The criticism is regaining the attention of the media and the regulators recently.

Searching Financial Times articles with key words “high frequency trading volatility” gives plenty of news, articles and blogs that argue for and against the claim. There is articles support the claim and point out that empirical market volatility has actually gone down. Such articles include ‘Markets: In search of a fast buck’ on February 19th, 2013, ‘Comment: Some HFT myths debunked’ on March 1st, 2013 and ‘FBI joins SEC in computer trading probe’ on March 5th, 2013. However there are also articles such as ‘Scrutiny on high frequency traders’ on September 23rd, 2012, ‘High frequency traders’ claims refuted by studies’ on September 30th, 2012, ‘BoE on HFT: A large absolute noise contribution’ on December 3rd, 2012 reporting that HFT could increase the stock market volatility. There is another argument that may be the short term volatility has gone down but the longer term expectation has gone up because HFT creates herding behavior. This includes ‘Markets: With the volume down’ on February 11th, 2013. Based on above FT articles, we can see that the HFT and volatility is related to all sorts of issues including market stability, short term and long term expectations, herding behavior and even systemic risk.

There are many academic literatures about the impact of HFT on volatility and the findings so far seem to be mixed. Hasbrouck and Saar (2011) find that HFT decreases short-term volatility. Zhang (2010) argues that HFT increase volatility. Zhang (2010) finds that HFT is positively correlated with stock price volatility after controlling for firm fundamental volatility and other exogenous determinants of volatility. The paper reports stronger positive correlation among the top 3,000 stocks in market capitalization and among stocks with high institutional holdings. The positive correlation is stronger during periods of high market uncertainty. They also find that HFT is negatively related to the market’s ability to incorporate information about firm fundamentals into asset prices. Stock prices tend to overreact to fundamental news when high-frequency trading is at a high volume.

Evangelos and Satchit (2012) is concerned that the most of these papers make no distinction between the informative and uninformative components of volatility. They argue that this would be reason why the results of these papers are less conclusive. They analyse the intraday behavior of HFT and its impact on aspects of market quality such as liquidity, price discovery and excess volatility. They use transactions date set for four UK stocks, over the period of a randomly selected week. They find that the ‘passive’ HFTs follow a trading strategy consistent with market making and as such their trades have alternating signs and are independent of recent (ten-second) price changes. By contrast, ‘aggressive’ HFTs exhibit persistence in the direction of their trades and trade in line with the recent (ten-second) price trend. They also find that both higher price volatility and lower spreads cause HFT activity to increase. Finally, they use a tick time specification to examine the impact of HFT activity on information-based volatility and excess volatility. The result shows that while HFTs have a higher information-to-noise contribution ratio than non-HFTs, there are instances where this is accompanied by a large absolute noise contribution.

The joint report of Securities and Exchange Commission (SEC) and the Commodity Futures Trading Commission (CFTC), investigating the Flash Crash of May 6th, 2010, states “HFTs began to quickly buy and then resell contracts to each other – generating a hot-potato volume effect as the same positions were rapidly passed back and forth. Between 2:45:13 and 2:45:27, HFTs traded over 27,000 contracts, which accounted for about 49 per cent of the total trading volume, while buying only about 200 additional contracts net”. From this report, we could see that the regulators are concerned with two aspects of HFC. Trading strategy that generated a “hot-potato” volume effect and the fast trading which increased the speed and the number of the trades.

It seems that there exists self-selection bias that the trading strategies or the styles that are employed by the high frequency traders tend to be more volatile. This could be a reason why HFT is blamed to increase the volatility although more frequent trading itself does not. In general, HFT consists of two components, predetermined algorithmic trading strategy and frequent trading. Computerized automated trading system is an essential component of HFT. In most of literatures, the impacts from the two are not distinguished. This seems to create the controversial arguments. Careful reading of existing literatures reveals that most of the existing works focus on the empirical evidences and have less emphasis on the possibility that the increase or the decrease in the volatility might be due to statistical properties. The focus on empirical evidences would be one of reasons why it is hard to distinguish the impact of one from that of the other.

Therefore theoretical investigation would be useful. This paper theoretically investigates the relationship between more frequent trading and price volatility at asset and portfolio level with a stochastic price process. Therefore it could separate the impact of highly frequent trading component of HFT from algorithmic trading strategy component. Hence the contributions of this paper are clear.

1. It provides a theoretical model that could investigate the properties of HFT and brings the discussion to asset and portfolio level from overall stock market level.

2. It provides a theoretical support that the availability of more frequent trading reduces asset price volatility.

3. The paper investigates HFT in relation to Time Series Momentum hence adds to existing Time Series Momentum literatures.

4. It theoretically finds that the portfolio that employs technical analysis with moving average (MA hereafter) prices would have lower volatility when more frequent trading is available.

The primary question to ask in this paper is “Does more frequent trading increase the asset and portfolio price volatility, everything else held constant?” This is closely related to the issue of holding period / return measurement interval. This study investigates from a statistical point of view and provides some theoretical evidences using a simple stochastic model. The paper finds that the availability of more frequent trading does not necessarily increase the price volatility of an asset and a portfolio. Since more frequent trading has clear benefits, the paper argues that the regulators should focus on the trading behavioral aspect of HFT but be less concerned about the fast trading aspect.

There are number of points that should be raised regarding the scope and the implications of this paper. In order to keep the paper focussed, I address such out-of-scope issues and clearly state that this paper does not intend to tackle those issues. This would enhance the transparency of the implication of the result. First, this study does not intend to investigate any other claimed sins of HFT such as the use of dark pool, herding behavior etc. This paper intends to validate the claims that HFT increases or decreases the market volatility from a theoretical perspective based on a stochastic model. Second, although the theoretical investigation shows that HFT decreases the portfolio price volatility, it only means that the volatility does not increase in statistical perspective. Portfolio volatility could, however, increase due to other aspects of HFT such as manager trading more aggressively. But such increase is due to portfolio managerial behavior difference, not due to HFT itself. This paper concentrates to analyse the impact of more frequent trading only in order to make the implication transparent.

The rest of the paper is organized as follows. Section 2 provides the back ground of the study including trading rules definition and time series price momentum. Section 3 presents the impact of more frequent trading on asset price volatility. Section 4 the impact of more frequent trading on portfolio volatility, Section 5 discusses policy implication of the result and Section 6 concludes the paper.

2. Background

2.1 Trading Rules Definition

This paper employs and expands the definition of the MA oscillator by Brock, Lakonishok and LeBaron (1992), which states

> “According to the moving average trading rule, buy and sell signals are generated by two moving averages of the level of the index – a long-period average and a short-period average. In its simplest form this strategy is expressed as buying (or

> selling) when the short-period moving average rises above (or falls below) the long-period moving average. The idea behind computing moving averages it to smooth out an otherwise volatile series. When the short-period average exceeds the long-period moving average, a trend is considered to be initiated. A very popular moving average rule is 1-200, where the short period is one day and the long period is 200 days. While numerous variations of this rule are used in practice, we attempted to select several of the most popular ones: 1-50, 1-150, 5-150, 1-200, and 2-200."

Hong and Satchell (2013, HS hereafter) use this rule to explain autocorrelation amplification in technical analysis and this paper develops a modified version of HS model. Let *l* stands for the time period that the MA is computed for a long position and *s* for the time period that the MA is computed for a short position. Hence *s* > *l* and they are overlapping. Denote SMA as MA computed over time period *s* and LMA as MA computed over time period *l*. For example at time t, if *s* = 7 and *l* = 3, then SMA is computed over time t – 6 to t and LMA is computed over time t – 2 to t. In this paper, we classify two opposite MA rules with SMA and LMA, MAbull(*l*,*s*) and MAbear(*l*,*s*). and define that MAbull(*l*,*s*) rule takes a long position in the asset when SMA > LMA and taking no position in the asset when SMA < LMA and MAbear(*l*,*s*) rule does the opposite.

Definition 1: *The MA(l,s) position can be represented as*

$$MA(l,s) = \max\left(\delta(t), 0\right) \quad (1)$$

where $\delta(t) = \begin{cases} 1 \text{ if } LMA(t) - SMA(t) > 0 \\ 0 \text{ if } LMA(t) - SMA(t) < 0 \end{cases}$

and $SMA(t) = \dfrac{\sum_{i=1}^{s} p(t-i+1)}{s}$ and $LMA(t) = \dfrac{\sum_{i=1}^{l} p(t-i+1)}{l}$

where p(*t*) is an asset price at time *t*. The logic behind the MA rule is well known, when price penetrates the MA from the below, the bull trend is believed to be established and a trader wants to take advantage of this in an expectation that there will be further up movements in prices.

2.2 Time Series Price Momentum

This paper will investigate the volatility of price based momentum trading strategy, the MA rule. Empirical studies of momentum trading strategies report supporting evidence for the profitability, including Brock, Lakonishok and LeBaron (1992), Allen and Karjalainen (1999), and Lo, Mamaysky, and Wang (2000). These results suggest that the trading strategy beats the market in risk neutral terms, hence it is popular. Majority of existing momentum literatures focus on the relative performance of securities in the cross-section. HFT could involve more than one exchange and takes advantage of cross sectional price discrepancies. Such cross sectional analysis could be an interesting topic of research but is out of the paper's scope.

There is another type of momentum that could be investigated, time series momentum. Moskowitz, Ooi, and Pedersen (2012) argue "Time series momentum is the momentum within asset price process. It is related to but different from the phenomenon known as 'momentum' in the finance literature, which is primarily cross-sectional in nature. The momentum literature focuses on the relative performance of securities in the cross-section, finding that securities that recently outperformed their peers over the past three to 12 months continue to outperform their peers on average over the next month. Rather than focus on the relative returns of securities in the cross-section, time series momentum focuses purely on a security's own past return."

Time series momentum is often investigated along with technical analysis. Zhu and Zhou (2009) analyse the usefulness of technical analysis, specifically the MA trading rule from an asset allocation perspective. They show that, when stock returns are predictable, technical analysis adds value to commonly used allocation rules that invest fixed proportions of wealth in stocks. Bajgrowicz and Sxaillet (2012) investigates the apparent historical success of technical trading rules on daily prices of the Dow Jones Industrial Average index from1897 to 2011, and they use the false discovery rate (FDR) as a new approach to data snooping. This paper also investigates time series momentum and identifies the statistical impact of highly frequent trading scheme applied to price momentum trading strategy based on MAs.

3. Impact of the More Frequent Trading on Asset Volatility

Let de-meaned log price process $q(t) = \log p(t) - \mu t$ has distribution $q(t) \sim \left(0, \sigma^2\right)$. And denote the autocorrelation between $q(t)$ and $q(t+h)$ as $\rho_A(h)$. Assume the de-meaned log price process $q(t)$, follows the Ornstein Uhlenbeck process.

$$dq(t) = -\theta q(t)dt + \sigma dW(t) \quad (2)$$

where θ and σ are the parameters of the OU process. Furthermore, assume $\sigma > 0$, and $W(t)$ denotes a Wiener process. The OU process is often used to model mean reverting financial processes. Mean reverting processes are naturally attractive to model financial asset prices because of the economic argument that when prices are 'too high', demand will reduce, and supply will increase prices, producing a counter-balancing effect. When prices are 'too low' the opposite will occur, again pushing prices back towards some form of long term mean. This is the primary reason why I employ the OU process to the current de-meaned process, hence the process will revert to zero. These and more general stochastic differential equation systems have been analysed econometrically by Sargan (1974), Phillips (1974) and Robinson (1977).

Another reason for using the OU process is that the equations can be explicitly solved and there are exact solutions for discretized versions of this model. The use of this process has been heavily studied by Bergstrom. (1990). This process has been successfully used by many others including Lo and Wang (1995), Kramer and Richter (2007) and Onalan (2009) in studying financial asset price process. The analytical solution for this this process is well known.

$$q(t+h) = e^{-\theta(t+h)}q(0) + \sigma \int_0^{t+h} e^{-\theta(t-u)} dW(u) \quad (3)$$

h is the time interval that price is reported hence is the minimum time interval that a trading could occur. I call *h* the minimum trading interval. *h* could be seen as a return measurement interval because it is the minimum time interval that returns could be measured. This links the current paper to existing intervalling effect literatures such as Dimson (1979), Corhay (1992) and Handa, Kothari, and Wasley (1993). Equation (4) has an exact AR(1) representation.

$$q(t+h) = e^{-\theta h}q(t) + \sigma \int_t^{t+h} e^{-\theta(t+h-u)} \cdot dE(u) \quad (4)$$

where subscript A indicates asset. The subscript A is included to make notational distinction of asset return from trading strategy return, which will be introduced later. The means and variances of $q(t)$ can be written as

$$E\left(q(t+h)\right) = e^{-\theta(t+h)}q(0), Var\left(q(t+h)\right) = \sigma_q^2(h) = \left(\frac{1-e^{-2\theta(t+h)}}{2\theta}\right)\sigma^2$$

Note that the implicit assumption to keep the variances positive is that the de-meaned price process is mean reverting or $\theta > 0$. This is a reasonable assumption. I next calculate time t conditional variance and covariance of de-meaned price.

$$Cov\left[q_A(t), q_A(t+h)\right] = \left(\frac{1-e^{-2\theta t}}{2\theta}\right)\sigma^2 e^{-\theta h} \quad (5)$$

Therefore the autocorrelation of asset return can be expressed as

$$Corr\left[q_A(t), q_A(t+h)\right] = \rho_A(h) = e^{-\theta h} \quad (6)$$

Proposition 1: *The sensitivity of variance of q(t) with respect to h is*

$$\frac{\partial \sigma_q^2(h)}{\partial h} = \sigma^2 e^{-2\theta(t+h)} > 0 \quad (7)$$

Conditioning on time zero, Proposition 1 has two important implications. [1] It shows that smaller minimum trading interval or the availability of more frequent trading reduces the asset volatility. This is consistent with the common belief of the financial industry. Statistically speaking, HFT could reduce the volatility. [2] As the measurement interval, *h*, approaches to zero, the asset volatility collapses to zero

4. Impact of the More Frequent Trading on Portfolio

4.1 Technical Trading Rule Model

This section applies one of the most popular trading rules based on technical analysis, the MA rule, on the previously investigated price process to analyze the impact of shorter trading interval on portfolio volatility. This would be meaningful because the trading strategy or the algorithmic automated trading part of the HFT is held constant. This effectively controls any other aspects of HFT than the trading frequency and allows investigating the impact on volatility from shorter trading interval only.

As previously noted the MAs are defined as

$$SMA(t) = \frac{\sum_{i=1}^{s} p(t-i+1)}{s} \quad \text{and} \quad LMA(t) = \frac{\sum_{i=1}^{l} p(t-i+1)}{l} \quad \text{where} \quad s > l$$

The model used in this section is a modified version of the model built in HS.

Definition 2: *The critical value of a MA(s,l)) rule is defined as*

$$C(s,l;t) = \frac{LMA(t)}{SMA(t)} = \exp\left(w'Q(t)\right) \quad (8)$$

where $w' = \left(\left(\frac{1}{l} - \frac{1}{s}\right) \quad \cdots \quad \left(\frac{1}{l} - \frac{1}{s}\right) \quad -\left(\frac{1}{s}\right) \quad \cdots \quad -\left(\frac{1}{s}\right)\right)$ and
$Q(t)' = \left(q(t) \quad q(t-1) \quad \cdots \quad q(t-l+1) \quad \cdots \quad q(t-s+1)\right)$

The derivation of definition 2 is in Appendix A2 of HS. The strategy takes a long position if *C*(s,l;t) becomes more than 1 and liquidate the position if *C*(s,l;t) becomes less than 1. As noted in HS, when the MA rule is applied based on a positive trending price process, the exit point (where the manager liquidates a long position due to the current price penetrating the MA from above) will always be higher than the entry point hence the return is always positive. However, the manager is simply picking up the price trend and the same return can also be made with a buy and hold strategy. This is less interesting. In order to separate the impact of momentum from the market-wide trend, I use the de-meaned log price, which also has useful statistical properties for estimation. The 'de-meaned' MA rule also has a simple price interpretation as shown below.

Assuming stationarity in the second moments, the bivariate distribution of C(t) and C(t-1) can be expressed as,

$$\begin{pmatrix} w'Q(t) \\ w'Q(t-m) \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} w'\Omega w & w'\Omega_m w \\ w'\Omega_m w & w'\Omega w \end{pmatrix} \right) \quad (9)$$

where Ω is the ($s \times s$) covariance matrix of Q(t) and Ω_C is ($s \times s$) covariance matrix between Q(t) and Q(t-1). For now, we do not assume anything about the covariance matrix but simply note that the autocovariance is not zero. Lemma 1 of HS shows that the MA (l,s) rule based on a de-meaned log price is equivalent to buying a unit of an asset when

$$P(t) > \exp\left(\frac{\mu m}{2}\right)\left(\prod_{j=1}^{m-1} P(t-j)\right)^{\frac{1}{m-1}} \quad (10)$$

Under the current assumptions, $C(s,l;t)$ has a log normal distribution.

$$C(s,l;t) = \exp(w'Q(t)) \;\; w'Q(t) \sim N(0, w'\Omega w)$$

Let $\rho_A(k)$ be the asset price autocorrelation in log prices between k periods. And let $\rho_A(0) = 1$ and $\rho_A(k) = \rho_A(-k)$ since we assume stationarity in second moments. Note that Ω is a covariance matrix of Q(t), which is a vector of q(i) values (i = 1 … t). Ω_m is a covariance matrix between Q(t) and Q(t-1)

In Proposition 3 of HS, the variance for C(s,l;t) and the covariance between C(s,l;t) and C(s,l;t-m), when l = 0, is derived as

$$\sigma_T^2 = \sigma_q^2 \left(\begin{array}{l} a^2\rho_A(m) + b^2\left((s-1)\rho_A(m) + \sum_{i=1}^{s-2}(s-i-1)\left(\rho_A(m+i) + \rho_A(m-i)\right)\right) \\ + ab\left(\sum_{i=0}^{s-2}\rho_A(m+i+1) + \sum_{i=0}^{s-2}\rho_A(m-i-1)\right) \end{array} \right) \quad (11)$$

where $a = \left(\frac{1}{l} - \frac{1}{s}\right)$ and $b = -\left(\frac{1}{s}\right)$

The proof is in Appendix A6 of HS and I include the proof in Appendix A1 of this paper since HS has not been published yet. l is assumed to be 1 for calculation simplicity. This assumption would not alter the mathematical result and is a practical one since today's price is often compared against historical moving average price when the MA rule is used. When the MA rule is practically applied, managers often take the shorter time period as 1 (l =1). When l = 1, it is referred to as the "MA crossover rule". This case can be treated as a special case since it makes the equation significantly simpler by getting rid of the first of both double summation terms.

4.2 Impact on a Portfolio of Technical Trading Strategy

This section applies the OU model of de-meaned price process to the technical trading rule model and investigates the variance of the trading strategy with respect to the minimum trading interval, h. Here h is the interval that prices are reported. Therefore the interval that SMA is computed, s, varies proportionate to the size of h. For 10 day MA and when h is 1 week, s = 2, when h is 1 day, s = 10, when h is 1 hour, s = 65, when h is 1 minute, s = 3900 and when h is 1 second, s = 234000. Therefore s depends on the size of h and s should be denoted as $s(h)$ to note its dependency on h. More specifically, given a constant length of s, s', $s(h)$ could be modelled as $s(h) = s'/h$. Here, s' could have any time unit, nano-second, second, minute, hours, day etc., as long as it is consistent with the time unit of h. Since s is dependent on h, a and b becomes dependent on h as well. h could take any positive non-zero value up to 10 days in this example

Remark 1: *The variance of the technical trading strategy critical value, when the underlying asset price follows the OU model and assuming l = 1, can be written as*

$$\sigma_T^2(h) = \sigma^2\left(\frac{1-e^{-2\theta(t+h)}}{2\theta}\right)\left(\begin{aligned} & a(h)^2 + b(h)^2\left(\left(s(h)-1\right) + \sum_{i=1}^{s(h)-2}\left(s(h)-i-1\right)\left(e^{-\theta i}+e^{\theta i}\right)\right) \\ & + a(h)b(h)\left(\sum_{i=0}^{s(h)-2} e^{-\theta(i+1)} + \sum_{i=0}^{s(h)-2} e^{\theta(i+1)}\right)\end{aligned}\right) \quad (12)$$

where $a(h) = \left(1 - \frac{1}{s(h)}\right)$ and $b(h) = -\left(\frac{1}{s(h)}\right)$

The proof of Remark 1 is in Appendix A2.

Proposition 2: *The sensitivity of variance of the trading strategy critical value, C(t), with respect to h is positive*

$$\frac{\partial \sigma_T^2(h)}{\partial h} > 0 \quad (13)$$

The proof of Proposition 2 is in Appendix A3. Proposition 2 indicates when de-meaned asset price follows a mean-reverting process, a portfolio that employ technical analysis and take advantage of time series momentum have smaller portfolio price or level volatility as more frequent trading becomes available. Note that the volatility here is the level volatility against to the return volatility.

It is often argued that HFT firms are market makers and provide liquidity to the market which has lowered volatility and helped narrow Bid-offer spreads, making trading and investing cheaper for other market participants. This paper adds another reason why HFT activities would lower the price volatility, hence contributing to the existing literatures of HFT. This result does make any indication about the effect of the automated trading system on volatility. The trading strategy is controlled. It is more likely to increase the volatility if they are programmed to trade more aggressively than the average traders in the market.

These opposing impacts would cause such controversial arguments regarding the impact of HFT on volatility. This paper shows that, at least from the statistical point of view, more frequent aspect of HFT reduces the price volatility.

4.3 Influence of Time Series Momentum

Equation (12) can be decomposed into three factors.

Remark 2: *The variance of the technical trading strategy critical value, when the underlying asset price follows the OU model and assuming l = 1, can be decomposed into three components*

Term 1: $\sigma_A^2 a(h)^2 \quad (14)$

Term2: $\sigma_A^2 b(h)^2 \left(\left(s(h)-1\right) + \sum_{i=1}^{s(h)-2} \left(s(h)-i-1\right)\left(e^{-\theta i} + e^{\theta i}\right) \right) \quad (15)$

Term 3: $\sigma_A^2 a(h) b(h) \left(\sum_{i=0}^{s(h)-2} e^{-\theta(i+1)} + \sum_{i=0}^{s(h)-2} e^{\theta(i+1)} \right) \quad (16)$

The proof of Remark 2 is omitted. Term 1 is the volatility induced from the short term MA. Since l = 1, the short term MA is the current price. Term 2 is the volatility due to the long term MA and Term 3 is the interaction term. They are all proportionate to the asset price volatility, which indicate that all components of the trading strategy volatility are based on the volatility of the underlying asset. Term 1 and 2 are positive while Term 3 is negative. From equation (6) we find that Term 2 includes sum of autocorrelations from first lag to ($s(h)$ – 2) th lag. Term 3 also includes sum of autocorrelations from first lag to ($s(h)$ – 1) th lag. This indicates that the covariance structure of lagged de-meaned prices influences the current variance of the trading strategy. This makes logical sense since the trading strategy employs two MAs with different lengths, hence taking advantage of time series momentum. Note that for large $s(h)$, The absolute value of Term 2 is larger than the absolute value of Term 3.

Remark 2 indicates that the effect of time series momentum is cumulative in the current trading strategy volatility. The sum of autocorrelation is the magnitude of time series momentum. If all lagged covariances of $q(t)$ and $q(t+h)$ are positive but diminishing as the order of lag increases, this mean the impact of a shock on price is consistent but diminishes over time. There is no price reversal and the sum of autocorrelation would be equivalent to the sample Ljung – Box statistics with the sample lag of $s(h)$ – 1 or $s(h)$ – 2. In this case, the trading strategy volatility is positively related with the length of the longer period MA, s'. If there exist price reversal as many behavioral finance literatures report, then the evolution of the trading strategy volatility with respect to s' would not be monotonic. As previously explained s' is different from $s(h)$. The unit of s' is time, it can be seconds, minutes, hours, days etc. The unit of $s(h)$ is the minimum interval that the price is quoted, h. For various h, s' is constant while $s(h)$ is variant.

5. Policy Implications

This paper builds a simple model when the asset price follows a stochastic process to investigate the price volatilities of the asset and the portfolio that employs technical analysis. The paper finds that the volatility falls as the minimum trading interval decreases. This could imply that the claimed increase in volatility might be due to the behavioral difference of the traders rather than more frequent trading. Although the theoretical result is consistent with what many traders claim, it is also consistent with the FT article, 'Markets: With the volume down', which argue that may be the short term volatility has gone down but the longer term expectation has gone up because HFT creates herding behavior. It does not necessarily be the herding behavior only. It could be self-selection bias of traders that traders who employ more volatile trading strategies prefer HFT. Investigating and comparing the behavior of high frequency traders with the rest would be useful and this could be an interesting future research topic.

The result of this paper suggests that in regulating HFT activities, the more frequent trading aspect of the HFT should be separated from algorithmic code that determines the trading behavior. While fast and more frequent trading has clear benefits of lowering the cost of trading, increasing the informativeness of quotes, providing liquidity and eliminating arbitrage, the result suggests that there is no theoretical evidence that it increases the price volatility.

Flash Crash of May 6th, 2010 could be a good example for this. The U.S. SEC and CFTC concluded that the cause was a single sale of \$4.1 billion in futures contracts by a mutual fund, identified as Waddell & Reed Financial, in an aggressive attempt to hedge its investment position. Such large sales created a selling pressure in the market and the pre-coded algorithmic trading systems amplified the selling pressure, which was transferred from the futures markets to the stock market by arbitrageurs who started to buy the cheap futures contracts but sell cash shares on markets like the New York Stock Exchange. The computerized trading system on the stock market shut down as the sharp rise in buying and selling activities were detected.

This led to the abrupt drop in prices of individual stocks and other financial instruments. From the SEC/CFTC report itself:

> The combined selling pressure from the sell algorithm, HFTs, and other traders drove the price of the E-Mini S&P 500 down approximately 3% in just four minutes from the beginning of 2:41 pm through the end of 2:44 pm. During this same time cross-market arbitrageurs who did buy the E-Mini S&P 500, simultaneously sold equivalent amounts in the equities markets, driving the price of SPY (an exchange-traded fund which represents the S&P 500 index) also down approximately 3%.
>
> Still lacking sufficient demand from fundamental buyers or cross-market arbitrageurs, HFTs began to quickly buy and then resell contracts to each other – generating a "hot-potato" volume effect as the same positions were rapidly passed back and forth. Between 2:45:13 and 2:45:27, HFTs traded over 27,000 contracts, which accounted for about 49 per cent of the total trading volume, while buying only about 200 additional contracts net

This extraordinary event was due to the algorithmic or automated feature, not high frequency feature of HFT. The selling pressure due to the lack of liquidity (demand) was the reason for the crash. The hot-potato volume effect and HFT taking 49 per cent of trading volume were due to the herding effect created by algorithms that acted in the same style simultaneously. This is not due to more frequent trading. The lack of liquidity was induced because the trading algorithms were coded in similar style, which created the herding behavior. This has not much to do with fast trading. The crash happened between 2:45:13 and 2:45:27, took 14 seconds. This is due to fast trading. Even if fast trading would have not been available, the same crash would have happened as long as the algorithms acted as they were coded. Of course, it could be argued that traders would have prevented the trade if this took a longer time. But then it could be also argued that traders could have programmed the trading strategies accordingly. Again the debate brings us back to the traders' behavior and trading algorithms, not more frequent trading.

From the last statement of the quoted report, we find that the regulators are concerned with fast and frequent trading. The result of this paper suggests that the regulators who are concerned with rise in volatility due to HFT should not focus how fast or frequent trades occur. They should concentrate their regulatory effort to find and control which aspects of the trading algorithms created such behavior. In short, the regulators who are concerned with the volatility induced by HFT should concentrate regulatory effort on the behavior and the characteristics of high frequency traders rather than on how frequent they trade.

6. Conclusion

Whether HFT increases volatility or not is a very controversial issue. Traders often claim that HFT reduces the volatility. The critiques claim the opposite. Regulators seem to be concerned with both algorithmic of trading strategy and the fast frequent trading. Most of existing literatures investigate the issue empirically. Since algorithmic trading is a necessary condition for fast trading, empirical investigation would not be able to distinguish the impact of one from that of the other.

This paper theoretically investigates asset and portfolio level volatility when more frequent trading is available, using the Ornstein-Uhlenbeck model.

The paper separates the effect of trading strategy difference on volatility from the effect from fast frequent trading. The paper finds that for an asset that the de-meaned asset price process follows the OU process and for a portfolio that employ the moving average rule based on such asset, availability of HFT decreases both asset and portfolio price volatility.

The contributions of this paper can be summarized as below. 1. It provides a theoretical model that could investigate the properties of HFT and brings the discussion to asset and portfolio level from overall stock market level. 2. It provides a theoretical support that the availability of more frequent trading reduces asset price volatility. 3. The paper investigates HFT in relation to Time Series Momentum hence adds to existing Time Series Momentum literatures. 4. It theoretically finds that the portfolio that employs technical analysis with moving average prices would have lower volatility when more frequent trading in available

The paper implies that one should separate the impact of more frequent trading from that of automated trading system in terms of regulating HFT activities since more frequent trading does not necessarily induce additional volatility.

7. Reference

Allen, F., and R. Karjalainen. 1999. “Earnings and price momentum.” *Journal of Financial Economics* 51(3): 245–271.

Atkins, R. 2013. “Markets: With the volume down”. Financial Times [online], 11 February. Available from: <http://www.ft.com> [Accessed 3 April 2013].

Bajgrowicz, P., and O. Sxaillet. 2012. “Size, value, and momentum in international stock returns”. *Journal of Financial Economics* 106: 473–491.

Bergstrom, A. R. 1990. *Continuous Time Econometrics Modelling*. Oxford University Press: Oxford

Brock, W., Lakonishok, J., LeBaron, B., 1992. Simple Technical Trading Rules and the Stochastic Properties of Stock Returns. *The Journal of Finance* 47(4): 1731 – 1764.

Corhay, A. 1988. “The adjustment for the intervalling effect bias in beta: A broader and multiperiod test”. *European Institute for Advanced Studies in Management, Brussels, Working paper* 88-26

Dimson, E. 1979. “Risk measurement when shares are subject to infrequent trading”. *Journal of Financial Economics* 7(2): 197-226

Evangelos B., and S. Satchit S. 2012. “High-frequency trading behaviour and its impact on market quality: evidence from the UK equity market”. Bank of England Working Paper No. 469

Handa, P., S. P. Kothari, and C. Wasley. 1993. “Sensitivity of Multivariate Tests of the Capital Asset-Pricing Model to the Return Measurement Interval”. *Journal of Finance* 48(4): 1543-51

Hasbrouck, J. and G. Saar. 2011. “Low-Latency Trading”, *working paper*

Heaney, V. 2012. “High frequency traders’ claims refuted by studies”. Financial Times [online], 30 September. Available from: <http://www.ft.com> [Accessed 3 April 2013].

Hendershott, T., Jones, C. M., and A. J. Menkveld. 2010. Does Algorithmic Trading Improve Liquidity?. *The Journal of Finance* 66(1): 1-33

Hong K., and S. Satchell. 2013. “Time Series Momentum Trading Strategy and Autocorrelation Amplification”, *working paper*

Kaminska, I. 2012. “BoE on HFT: “A large absolute noise contribution”. Financial Times [online], 3 December. Available from: <http://www.ft.com> [Accessed 3 April 2013].

Kramer, R., and M. Richter. 2007. “A generalized bivariate Ornstein-Uhlenbeck model for financial assets”, Tagungsband zum Workshop 'Stochastische Analysis', Chemnitz

Lenterman, R. 2013. “Comment: Some HFT myths debunked”. Financial Times [online], 1 March. Available from: <http://www.ft.com> [Accessed 3 April 2013].

Lo, A. W., H., Mamaysky, J. Wang. 2000. “Foundations of Technical Analysis: Computational Algorithms, Statistical Inference, and Empirical Implementation”. *Journal of Finance* 55(4): 1705–1765.

Lo, A. W., and J. Wang. 1995. “Implementing Option Pricing Models When Asset Returns Are Predictable”. *Journal of Finance* 50(1): 87-129

Massoudi A., and M. Mackenzie. 2013. “Markets: In search of a fast buck”. Financial Times [online], 19 Feburary. Available from: <http://www.ft.com> [Accessed 3 April 2013].

Moskowitz T., Y. H. Ooi, L.H. Pedersen. 2012. “Time Series Momentum”. *Journal of Financial Economics* 104(2): 228–250.

Onalan, O. 2009. “*Financial Modelling with Ornstein – Uhlenbeck Processes Driven by Levy Process*”, Proceedings of the World Congress on Engineering, 2009, Vol II.

Phillips, P.C.B. 1974. “The estimation of some continuous time models”, *Econometrica*(42): 803-823

Robinson, P. M. 1977. “The construction and estimation of continuous time models and discrete approximations in econometrics”. *Journal of Econometrics* 6(2): 173-197

Sargan, J. D. 1974. “Some discrete approximations to continuous time stochastic models”, *Journal of the Royal Statistical Society* B 36: pages 74-90

Scannell, K. 2013. “FBI joins SEC in computer trading probe”. Financial Times [online], 5 March. Available from: <http://www.ft.com> [Accessed 3 April 2013].

U.S. Securities and Exchange Commission and the Commodity Futures Trading Commission. 2010. “Findings Regarding the Market Events of May 6, 2010”. [online], 10 Octover. Available from: < http://www.sec.gov/news/studies/2010/marketevents-report.pdf> [Accessed 5 April 2013].

Skypala, P. 2012. “Scrutiny on high frequency traders”. Financial Times [online], 23 September. Available from: <http://www.ft.com> [Accessed 3 April 2013].

Zhang F., 2010. “The Effect of High-Frequency Trading on Stock Volatility and Price Discovery”, *working paper*

Zhu, Y., and G. Zhou. 2009. “Technical analysis: An asset allocation perspective on the use of moving averages”. *Journal of Financial Economics* 92(3): 519–544.

7 Appendix

A1. Proof of Proposition of HS

Note that this is Appendix A6 of HS. Decompose Ω into four pieces where Ω_{11} is $l \times l$, Ω_{12} is $l \times (s-l)$, Ω_{21} is $(s-l) \times l$ and Ω_{22} is $(s-l)\times(s-l)$, also decompose $w' = (w_1' \quad w_2')$ where w_1' is $1 \times s$ and w_1' is $1 \times (s-l)$

Let $a = \left(\frac{1}{l} - \frac{1}{s}\right)$ and $b = \left(\frac{1}{s}\right)$

$$\underset{l \times l}{\Omega_{m11}} = \sigma^2 \begin{bmatrix} \rho(m) & \rho(m+1) & \cdots & \rho(m+l-1) \\ \rho(m-1) & \rho(m) & \cdots & \rho(m+l) \\ & & \ddots & \vdots \\ \vdots & & \rho(m) & \\ & & & \ddots \\ \rho(m-l+1) & & \cdots & \rho(m) \end{bmatrix}$$

$$\underset{l \times (s-l)}{\Omega_{m12}} = \sigma^2 \begin{bmatrix} \rho(m+l) & \rho(m+l+1) & \cdots & \rho(m+s-1) \\ \rho(m+l-1) & \rho(m+l) & \cdots & \rho(m+s-2) \\ \vdots & & & \vdots \\ & & \ddots & \\ \rho(m+1) & & \cdots & \rho(m+s-l) \end{bmatrix}$$

$$\underset{(s-l) \times l}{\Omega_{m21}} = \sigma^2 \begin{bmatrix} \rho(m-l) & \rho(m-l+1) & \cdots & \rho(m-1) \\ \rho(m-l+1) & \rho(m-l) & & \\ \vdots & \vdots & \ddots & \vdots \\ \rho(m-s+1) & \rho(m-s+2) & \cdots & \rho(m-s+l) \end{bmatrix}$$

$$\underset{(s-1) \times (s-1)}{\Omega_{m22}} = \sigma^2 \begin{bmatrix} \rho(m) & \rho(m+1) & \cdots & \rho(m+s-l-1) \\ \rho(m-1) & \rho(m) & \cdots & \rho(m+s-l-2) \\ & & \ddots & \vdots \\ \vdots & & \rho(m) & \\ & & & \ddots \\ \rho(m-s+l-1) & & \cdots & \rho(m) \end{bmatrix}$$

$$\Omega_m = \begin{bmatrix} \Omega_{m11} & \Omega_{m12} \\ \Omega_{m21} & \Omega_{m22} \end{bmatrix}, \quad w_1' = (a \quad \cdots \quad a) \text{ and } w_2' = (b \quad \cdots \quad b)$$

$$w_1'\Omega_{m11}w_1 = a^2\sigma^2\left(l\rho(m) + \sum_{i=1}^{l-1}\left(\rho(m+i) + \rho(m-i)\right)\right)$$

$$w_2'\Omega_{m22}w_2 = b^2\sigma^2\left((s-l)\rho(m) + \sum_{i=1}^{s-l-1}(s-i-1)\left(\rho(m+i) + \rho(m-i)\right)\right)$$

$$w_1'\Omega_{m12}w_2 = ab\sigma^2\left(\sum_{j=0}^{l-1}\sum_{i=0}^{s-l-1}\rho(m+l+i-j)\right)$$

$$w_2'\Omega_{m21}w_1 = ab\sigma^2\left(\sum_{j=0}^{l-1}\sum_{i=0}^{s-l-1}\rho(m-l-i+j)\right)$$

$$w'\Omega_m w = w_1'\Omega_{m11}w_1 + w_1'\Omega_{m12}w_2 + w_2'\Omega_{m21}w_1 + w_2'\Omega_{m22}w_2$$

$$= \sigma^2\left(\begin{array}{l} a^2\left(l\rho(m) + \sum_{i=1}^{l-1}\left(\rho(m+i) + \rho(m-i)\right)\right) \\ + b^2\left((s-l)\rho(m) + \sum_{i=1}^{s-l-1}(s-i-1)\left(\rho(m+i) + \rho(m-i)\right)\right) \\ + ab\left(\sum_{j=0}^{l-1}\sum_{i=0}^{s-l-1}\rho(m+l+i-j) + \sum_{j=0}^{l-1}\sum_{i=0}^{s-l-1}\rho(m-l-i+j)\right) \end{array}\right)$$

A2. Derivation of Remark 1

The autocovariance of the technical trading strategy is

$$\sigma_q^2\begin{pmatrix} a^2\rho_A(m)+b^2\left((s-1)\rho_A(m)+\sum_{i=1}^{s-2}(s-i-1)\left(\rho_A(m+i)+\rho_A(m-i)\right)\right) \\ +ab\left(\sum_{i=0}^{s-2}\rho_A(m+i+1)+\sum_{i=0}^{s-2}\rho_A(m-i-1)\right) \end{pmatrix}$$

The variance of the technical trading strategy can be computed as the autocovariance when there is no lag, hence m = 0 in equation (). Substituting m = 0 and applying $\rho_A(0)=1$, we get

$$\sigma_q^2\begin{pmatrix} a^2+b^2\left((s-1)+\sum_{i=1}^{s-2}(s-i-1)\left(\rho_A(i)+\rho_A(-i)\right)\right) \\ +ab\left(\sum_{i=0}^{s-2}\rho_A(i+1)+\sum_{i=0}^{s-2}\rho_A(-i-1)\right) \end{pmatrix}$$

Given $Var\left(q(t)\right)=\sigma_q^2=\left(\frac{1-e^{-2\theta(t+h)}}{2\theta}\right)\sigma^2$ and $Corr\left[q_A(t),q_A(t+h)\right]=\rho_A(h)=e^{-\theta h}$, we get

$$\sigma_T^2=\sigma^2\left(\frac{1-e^{-2\theta(t+h)}}{2\theta}\right)\begin{pmatrix} a^2+b^2\left((s-1)+\sum_{i=1}^{s-2}(s-i-1)\left(e^{-\theta i}+e^{\theta i}\right)\right) \\ +ab\left(\sum_{i=0}^{s-2}e^{-\theta(i+1)}+\sum_{i=0}^{s-2}e^{\theta(i+1)}\right) \end{pmatrix}$$

Denoting that a, b and s are dependent on h, we have

$$\sigma_T^2(h)=\sigma^2\left(\frac{1-e^{-2\theta(t+h)}}{2\theta}\right)\begin{pmatrix} a(h)^2+b(h)^2\left(\left(s(h)-1\right)+\sum_{i=1}^{s(h)-2}\left(s(h)-i-1\right)\left(e^{-\theta i}+e^{\theta i}\right)\right) \\ +a(h)b(h)\left(\sum_{i=0}^{s(h)-2}e^{-\theta(i+1)}+\sum_{i=0}^{s(h)-2}e^{\theta(i+1)}\right) \end{pmatrix}$$

A3. Proof of Proposition 2

$$\sigma_T^2(h) = \sigma^2\left(\frac{1-e^{-2\theta(t+h)}}{2\theta}\right)\left(\begin{array}{l} a(h)^2 + b(h)^2\left(\left(s(h)-1\right) + \sum_{i=1}^{s(h)-2}\left(s(h)-i-1\right)\left(e^{-\theta i} + e^{\theta i}\right)\right) \\ + a(h)b(h)\left(\sum_{i=0}^{s(h)-2} e^{-\theta(i+1)} + \sum_{i=0}^{s(h)-2} e^{\theta(i+1)}\right) \end{array}\right)$$

where $a(h) = \left(1 - \frac{1}{s(h)}\right)$ and $b(h) = -\left(\frac{1}{s(h)}\right)$

We know that $\sigma_A^2(h) = \sigma^2\left(\frac{1-e^{-2\theta(t+h)}}{2\theta}\right) > 0$ and $\frac{\partial \sigma_A^2(h)}{\partial h} > 0$

Let $$\gamma(h) = \begin{array}{l} a(h)^2 + b(h)^2\left(\left(s(h)-1\right) + \sum_{i=1}^{s(h)-2}\left(s(h)-i-1\right)\left(e^{-\theta i} + e^{\theta i}\right)\right) \\ + a(h)b(h)\left(\sum_{i=0}^{s(h)-2} e^{-\theta(i+1)} + \sum_{i=0}^{s(h)-2} e^{\theta(i+1)}\right) \end{array}$$

We know that $\gamma(h) > 0$ and we assumed that $s(h) > 2$

More specifically,

$a(h) > 0$, $b(h) < 0$, $\sum_{i=1}^{s(h)-2}\left(s(h)-i-1\right)\left(e^{-\theta i} + e^{\theta i}\right) > 0$, $a(h)b(h) < 0$,
$\sum_{i=0}^{s(h)-2} e^{-\theta(i+1)} > 0$, $\sum_{i=0}^{s(h)-2} e^{\theta(i+1)} > 0$

$$\frac{\partial \gamma(h)}{\partial h} = \begin{array}{l} 2a(h)\frac{\partial a(h)}{\partial h} + 2b(h)\frac{\partial b(h)}{\partial h}\left(s(h)-1\right) + b(h)^2\frac{\partial s(h)}{\partial h} \\ + 2b(h)\frac{\partial s(h)}{\partial h}\left(\sum_{i=0}^{s(h)-2} e^{-\theta(i+1)} + \sum_{i=0}^{s(h)-2} e^{\theta(i+1)}\right) + b(h)^2\sum_{i=1}^{s(h)-2}\frac{\partial s(h)}{\partial h}\left(e^{-\theta i} + e^{\theta i}\right) \\ + \frac{\partial a(h)}{\partial h} b(h)\left(\sum_{i=0}^{s(h)-2} e^{-\theta(i+1)} + \sum_{i=0}^{s(h)-2} e^{\theta(i+1)}\right) + \frac{\partial b(h)}{\partial h} a(h)\left(\sum_{i=0}^{s(h)-2} e^{-\theta(i+1)} + \sum_{i=0}^{s(h)-2} e^{\theta(i+1)}\right) \end{array}$$

Since $\frac{\partial s(h)}{\partial h} > 0$, $\frac{\partial a(h)}{\partial h} > 0$, $\frac{\partial b(h)}{\partial h} > 0$,

$$\frac{\partial \gamma(h)}{\partial h} = \begin{bmatrix} 2a(h)\frac{\partial a(h)}{\partial h} + 2b(h)\frac{\partial b(h)}{\partial h}\left(s(h)-1\right) \\ + b(h)^2 \frac{\partial s(h)}{\partial h} + b(h)^2 \sum_{i=1}^{s(h)-2} \frac{\partial s(h)}{\partial h}\left(e^{-\theta i} + e^{\theta i}\right) \\ + \frac{\partial b(h)}{\partial h} a(h)\left(\sum_{i=0}^{s(h)-2} e^{-\theta(i+1)} + \sum_{i=0}^{s(h)-2} e^{\theta(i+1)} \right) \end{bmatrix} + \begin{bmatrix} 2b(h)\frac{\partial s(h)}{\partial h}\left(\sum_{i=0}^{s(h)-2} e^{-\theta(i+1)} + \sum_{i=0}^{s(h)-2} e^{\theta(i+1)} \right) \\ + \frac{\partial a(h)}{\partial h} b(h)\left(\sum_{i=0}^{s(h)-2} e^{-\theta(i+1)} + \sum_{i=0}^{s(h)-2} e^{\theta(i+1)} \right) \end{bmatrix}$$

$$\frac{\partial a(h)}{\partial h} = \frac{\partial b(h)}{\partial h} = s(h)\frac{\partial s(h)}{\partial h}$$

$$\frac{\partial b(h)}{\partial h} a(h)\left(\sum_{i=0}^{s(h)-2} e^{-\theta(i+1)} + \sum_{i=0}^{s(h)-2} e^{\theta(i+1)} \right) + 2b(h)\frac{\partial s(h)}{\partial h}\left(\sum_{i=0}^{s(h)-2} e^{-\theta(i+1)} + \sum_{i=0}^{s(h)-2} e^{\theta(i+1)} \right)$$
$$+ \frac{\partial a(h)}{\partial h} b(h)\left(\sum_{i=0}^{s(h)-2} e^{-\theta(i+1)} + \sum_{i=0}^{s(h)-2} e^{\theta(i+1)} \right) = \frac{\partial s(h)}{\partial h}\left(\sum_{i=0}^{s(h)-2} e^{-\theta(i+1)} + \sum_{i=0}^{s(h)-2} e^{\theta(i+1)} \right)\left(s(h)a(h) + 3b(h)\right) > 0$$

Therefore we have $\frac{\partial \gamma(h)}{\partial h} > 0$

Since $\sigma_T^2(h) = \sigma_A^2(h)\gamma(h)$

$$\frac{\partial \sigma_T^2(h)}{\partial h} = \frac{\partial \sigma_A^2(h)}{\partial h}\gamma(h) + \sigma_A^2(h)\frac{\partial \gamma(h)}{\partial h} > 0$$